ON MINIMUM INCOME, QUALIFICATION STRUCTURE AND SALARY SCALE

by

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Abstract

The consequences of imposing a minimum real income are studied within the framework of a general equilibrium model in which unemployment is compensated by transfers. With a disaggregated labor market, the equilibrium distribution of employment and real wages is characterized, depending on the existing qualification structure. The case of a rigid qualification structure in which there is no mobility between categories is first considered. Some flexibility is then introduced by assuming that the qualification structure is pyramidal in the sense that workers in a given category have access to categories corresponding to lower qualification levels. If a budget deficit results the existence of an equilibrium is however not guaranteed. While there is no general condition ensuring the absence of a deficit, the alternative offered by employment subsidies is shown to be compatible with full employment and a balanced budget under minimal assumptions.

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1. INTRODUCTION

The competitive equilibrium is often used as reference in the evaluation of economic policy in particular concerning unemployment. Beyond the pertinence of perfect competition, the validity of the assumptions which ensure the existence of a competitive equilibrium must be questioned. Among them the survival assumption is probably the most critical. There has been several versions of that assumption, the strongest one requiring that agents must have enough resources to survive *whatever the price system*.[1]

The problem raised by that assumption has been discussed by several authors including Georgescu-Roegen (1955); Koopmans (1957) and more recently Coles and Hammond (1994). The latest show that when the requirement that agents survive at equilibrium is not imposed the basic results of the theory of general competitive equilibrium remain valid, in particular the existence and the two welfare theorems. The fact that a portion of the population may disappear at equilibrium is therefore not *in itself* the consequence of a failure of the market economy. The introduction of some institutional mechanism by which the net real income of no agent falls below some predefined minimum is a possible solution. However a minimum income rule *alone* does not solve the problem as it may induce unemployment.

In the present paper we study the effect on employment and wages of introducing a minimum real income, while prices and (nominal) wages are otherwise perfectly flexible. We consider the case where compensations are paid to unemployed workers and financed by an income tax. Together with the minimum real income, this induces a *minimum real wage*.

We consider an economy with an aggregate consumption good, labor and money, and with two aggregate sectors, a production sector and a consumption sector.[2] A special attention is given to labor heterogeneity expressed in terms of differences in the qualification levels of the workers. Two kinds of qualification structures are considered. We first study the case where the qualification structure is rigid in the sense that workers in a given category cannot move to an other category. We then allow for some flexibility by assuming that the qualification structure is pyramidal in the sense that

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[1] This is the Arrow-Debreu version (1954). Weaker assumptions have been proposed by McKenzie (1959), Debreu (1962) and Arrow and Hahn (1971) although they do not modify the basic nature of the problem.

[2] In this we extend the three-commodity model studied by Barro and Grossman (1976) and Malinvaud (1977).

workers in a given category can move to categories corresponding to lower qualification levels. A simple characterization of an equilibrium with unemployment is given *in real terms* for each qualification structure. In the case of a rigid qualification structure the emerging salary scale is particular in the sense that workers belonging to categories in which unemployment prevails receive all the minimum real wage. In the other categories the real wages simply equal their marginal productivities. The extreme phenomenon by which the salary scale is compressed at the lower end does not necessarily arise when the qualification structure is flexible. In that case the wages in two successive categories are equal whenever unemployment prevails in the most qualified category. While unemployment may be lower when the qualification structure is more flexible there may be *under-employment* to the extent that some workers may have to accept jobs corresponding to lower qualifications.[3] To the extend that productivity is positively correlated to qualification, the social product will be lower as their effective productivity will be less than their potential one.

We study the effect of a change in the minimum real wage on the salary scale and on employment and its distribution. We show that while total employment and production always decrease following an increase in the minimum real wage, the effect on the real wage and employment distributions depends on the technological inter-relationships between the various types of labor. We then show that existence of a (short run) equilibrium depends on the capacity of the economy to finance the unemployment compensations from income taxes, without creation of money, a problem which is less acute under alternative redistribution schemes like for instance employment subsidies.

The paper is organized as follows. The structure of the economy is presented in section 2. The assumptions on the production and consumption sectors are introduced and the minimal real wage is defined on the basis of a minimum of subsistence, exogenously given. Section 3 studies the case of a rigid qualification structure. An equilibrium allowing for unemployment is defined and characterized in real terms. Section 4 extends the previous results to the case of a flexible qualification structure. The question of existence of an equilibrium is studied in section 5 and concluding remarks are offered in the last section.

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3 See Fitoussi (1994) for a comparative study on wage distributions in United states, United Kingdom and France.

2. THE STRUCTURE OF THE ECONOMY

2.1 The production sector

The economy is considered at a given period of time. There is one aggregate consumption good, n types of labor and money. The consumption goods are produced through a technology described by a production function $y = F(z_1 \dots z_n)$ where the n types of labor appear as arguments.[4] The production function is assumed to satisfy the following assumptions :

A.1 F is continuously differentiable, increasing and strictly concave, and $F(0) = 0$.

We denote by $F_j(z) = \partial F/\partial z_j$ the marginal productivity of type j labor. Labor is supplied *inelastically* and L_j stands for the total supply of type j labor, with $L_j > 0, j = 1 \dots n$. We denote by p the price level and by $w = (w_j)$ the vector of (nominal) wage rates. The demand for labor is derived from the maximization of profits $pF(z) - \Sigma_j w_j z_j$. The solution depends on the real wages, $\omega_j = w_j/p$, and is denoted by $z_j = z_j(\omega_1 \dots \omega_n)$. It results from the first order conditions :

(1) $$\text{for all } j,\ F_j(z) \leq \omega_j \quad \textit{with equality if } z_j > 0.$$

2.2 The consumption sector

The current income of the consumers includes wages, unemployment compensations and profits, net of taxes. It is allocated between present and future consumption. We denote by c the current demand for consumption goods and by M the amount of money transferred to the next period. For a given employment structure z, the resulting *aggregate* budget constraint is given by

(2) $$pc + M = M_o + (1-t)\,[\Sigma_j w_j z_j + \gamma \Sigma_j w_j (L_j - z_j)] + (1-\theta)\,[pF(z) - \Sigma_j w_j z_j]$$

where M_o is the initial stock of money, t and θ are the income tax rates, and γ is the proportion of the wage paid to unemployed workers. These parameters satisfy the following restrictions : $M_o > 0$, $0 \leq t < 1$, $0 \leq \theta \leq 1$ and $0 < \gamma \leq 1$.

The demand for consumption goods is given by the following demand function:

$$c = C(p,w,z)$$

which is defined for all (p,w,z) such that $p > 0$, $w \geq 0$ and $0 \leq z \leq L$. The demand for money is derived from the aggregate budget constraint (2). The parameters M_o, t, θ and

[4] Vectors are represented by bold characters, Σ_j represents the summation for j running from 1 to n, and vector inequalities follow the sequence $\geq$, $>$ and $>>$.

γ are implicit. The consumption function is assumed to be continuous in all its arguments, and to satisfy the following three conditions on its domain of definition:

A.2 $0 < pC(p,w,z) < M_o + (1-t)\,[\Sigma_j w_j z_j + \gamma \Sigma_j w_j (L_j - z_j)] + (1-\theta)\,[pF(z) - \Sigma_j w_j z_j]$,

A.3 $C(\alpha p, \alpha w, z) < C(p,w,z)$ *for all* $\alpha > 1$,

A.4 $C(\alpha p, \alpha w, z) \uparrow +\infty$ *when* $\alpha \downarrow 0$.

According to these assumptions, the demand for consumption goods and the demand for money are always positive. At *constant real wage*, the demand for consumption goods decreases following an increase in prices and wages while it tends to infinity when prices and wages tend to zero. Beyond the real balance effect, these assumptions imply restrictions on the underlying expectations held by consumers on future prices and wages similar to those introduced to ensure the existence of a temporary Walrasian equilibrium in a monetary economy.[5]

2.3 Minimum income rule

We start from a situation where there is no minimum income rule and no redistribution scheme i.e. $t = \theta = 0$ and $\gamma = 1$. In that case, the right hand side of the budget constraint (2) is given by $M_o + pF(L)$ and a *Walrasian equilibrium* is defined by a price level p^* and a wage vector w^* such that :

W.1 $p^* > 0$ *and* $C(p^*, w^*, L) = F(L)$,

W.2 *L maximizes* $[p^*F(z) - \Sigma_j w_j^* z_j]$ *subject to* $z \geq 0$.

The existence (and uniqueness) of a Walrasian equilibrium is easily shown to follow from our assumptions.[6] It is also a byproduct of the existence proof offered in section 5. By the assumption A.1 and the equilibrium condition W.2, the walrasian real wages coincide with the full employment marginal productivities i.e. $w_j^* / p^* = F_j(L)$, $j = 1 \ldots n$.

We now consider the introduction of a *minimum of subsistence* defined by a quantity e, $e > 0$, and expressed in terms of the aggregate consumption good. We assume that it is such that the real income of some workers falls below e at the Walrasian equilibrium. That means that the full employment marginal productivity of some type of labor is too low :

A.5 $F_j(L) < e$ *for some j*.

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[5] See Grandmont (1977).

[6] See Hildenbrand (1978) for a proof of existence of a Walrasian equilibrium in the three commodity model, allowing for an elastic labour supply.

The minimum of subsistence is however assumed to be feasible outside any particular institutional arrangement in the sense that, with the available technologies and resources, *survival of the all population is possible.* In our simple setting it means that it is possible to redistribute the full employment production so as to give at least the minimum of subsistence to all :

A.6 $e\,\Sigma_j L_j \leq F(L)$

Put differently that condition says that per capita productivity of the total labor force exceeds the minimum of subsistence.

Unemployment compensations are then introduced and income taxes are raised. As a consequence the minimum income rule induces the following constraint on nominal wages and prices :

$$\gamma(1\text{-}t)\, w_j \geq pe \quad \textit{for all } j,$$

It means that the net real income perceived by any agent never falls below e and it defines implicitly the *minimum real wage* :

(3) $$\omega_o = \frac{e}{\gamma(1\text{-}t)}$$

It ensures that net real labor incomes and unemployment compensations never fall below the minimum of subsistence.

3. EQUILIBRIUM WITH A RIGID QUALIFICATION STRUCTURE

3.1 Definition of an equilibrium

By rigid qualification structure we mean that labor cannot move between categories. Given ω_o, an *equilibrium* is defined by a price level $\bar{p}$, a wage vector $\bar{w}$ and an employment vector $\bar{z}$ such that :

E.1 $\bar{p} > 0$ *and* $C(\bar{p},\bar{w},\bar{z}) = F(\bar{z})$,

E.2 $\bar{z}$ *maximizes* $[\bar{p}F(z) - \Sigma_j \bar{w}_j z_j]$ *subject to* $z \geq 0$,

E.3 *for all* j, $\bar{w}_j \geq \bar{w}_o$ *and* $0 \leq \bar{z}_j \leq L_j$,

E.4 *for all* j, $\bar{z}_j < L_j$ *implies* $\bar{w}_j = \bar{w}_o$.

where $\bar{w}_o = \bar{p}\omega_o$ is the minimum nominal wage at the price level $\bar{p}$. Condition E.1 specifies that the equilibrium price level is positive and the consumption good market is in equilibrium. Condition E.2 specifies that the employment levels are derived from profit maximization. Following (1) condition E.2 implies that the real wage in any category of labor actually used equals its marginal productivity. Condition E.3 simply requires that the wage rates satisfies the minimum wage constraint while allowing for unemployment. Condition E.4 is the key condition. It says that if a type of labor is not fully employed, its wage rate must be equal to the minimum. Put differently, a wage rate above the minimum implies full employment of the corresponding type of labor.[7] This assumes that the competitive mechanism by which unemployment places a downward pressure on wages effectively works.

It is to be observed that at an equilibrium workers whose qualification is characterized by unemployment all receive the same wage i.e. the minimum nominal wage $\bar{w}_0$. Accordingly the unemployed workers all receive the same compensation i.e. the minimum gross income $\gamma \bar{w}_0$. This is a consequence of the fact that the qualification structure is rigid while prices and wages are flexible subject only to the constraint that real wages never fall below some minimum. We denote by $\bar{\omega}$ the vector of equilibrium real wages i.e. $\bar{\omega}_j = \bar{w}_j / \bar{p}$.

At an equilibrium unemployment compensations are paid and taxes are perceived, generating either a budget deficit or a budget surplus. *In real terms*, the deficit associated with the wage-employment pair $(\bar{\omega}, \bar{z})$ is given by :

$$\bar{d} = \gamma(1-t)\, \Sigma_j \bar{\omega}_j (L_j - \bar{z}_j) - t\, \Sigma_j \bar{\omega}_j \bar{z}_j - \theta\, [F(\bar{z}) - \Sigma_j \bar{\omega}_j \bar{z}_j].$$

Rearranging the terms at the right hand side and making use of the equilibrium condition E.4, the real deficit can be written as :

$$\bar{d} = e\, \Sigma_j (L_j - \bar{z}_j) + (\theta - t)\, \Sigma_j \bar{\omega}_j \bar{z}_j - \theta\, F(\bar{z}) \tag{4}$$

3.2 Characterization of an equilibrium

A careful examination of the equilibrium conditions shows that there is a *dichotomy* between the real part of the economy and its monetary counterpart. It is indeed possible to determine first the equilibrium real wages and employment levels, using the conditions E.2 and E.3, and then to proceed with the determination of the price level using condition E.1. This step is covered by the following proposition.

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[7] This type of condition characterizes the definition of equilibrium with price rigidities. See for instance Drèze (1975), Benassy (1975) or Dehez and Drèze (1982).

Proposition 1   Under the assumptions A.1, there exists *one and only one* pair $(\bar{\omega},\bar{z})$ such that for all j :

(i) $F_j(\bar{z}) \leq \bar{\omega}_j$ *with equality if* $\bar{z}_j > 0$,

(ii) $\bar{\omega}_j \geq \omega_o$ *and* $0 \leq \bar{z}_j \leq L_j$,

(iii) $\bar{z}_j < L_j$ *implies* $\bar{\omega}_j = \omega_o$.

Condition *(i)* is the maximization of *real* profit and is equivalent to the equilibrium condition E.2. Conditions *(ii)* and *(iii)* cover the equilibrium conditions E.3 and E.4. There is actually a continuum of pairs $(\bar{\omega},\bar{z})$ which satisfy the conditions *(i)* and *(ii)*. It is condition *(iii)* which provides uniqueness.[8]

Proof. The proof of proposition 1 offers a simple characterization of an equilibrium. Indeed the conditions *(i)* to *(iii)* coincide with the Kuhn-Tucker conditions associated with the maximization of the real profit, i.e. the profit *computed at the minimum real wage*, under the constraint of available employment :

(5) $$\text{Max } [F(z) - \omega_o \Sigma_j z_j] \quad \text{subject to } \; 0 \leq z_j \leq L_j \quad (j = 1...n).$$

This optimization program has a solution, by continuity of the objective function and compactness of the constraint set. Strict concavity of the production function implies uniqueness of the solution which is denoted by $\bar{z}$. Concavity of the objective function and convexity of the constraint set ensure that the Kuhn-Tucker conditions are necessary and sufficient : there exists a vector of non-negative Lagrange multipliers $\lambda = (\lambda_j)$ such that for all j :

$$\lambda_j > 0 \text{ implies } \bar{z}_j = L_j$$

and

$$F_j(\bar{z}) \leq \omega_o + \lambda_j, \text{ with equality whenever } \bar{z}_j > 0.$$

Proposition 1 then follows by defining the real wage rate for type j labor by :

$$\bar{\omega}_j = \omega_o + \lambda_j \quad (j = 1...n).$$

Indeed the Lagrange multipliers are all non-negative and unemployment of type j labor implies $\lambda_j = 0$. □

It is worth noticing that the parameters γ and t determine the real wage-employment pair $(\bar{\omega},\bar{z})$ through the minimum real wage, as defined by equation (3), independently of θ.

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[8] It can be shown that the proposition remains valid in the polar cases where the types of labor are either *perfect complements* or *perfect substitutes*.

The next step is to define the equilibrium prices. A price level $\bar{p}$ defines an equilibrium associated with the real wage-employment pair $(\bar{\omega},\bar{z})$ if demand and supply are equal on the consumption goods market :

$$C(\bar{p},\bar{p}\bar{\omega},\bar{z}) = \bar{y}.$$

Given a price system $\bar{p}$ satisfying that condition, $(\bar{p},\bar{p}\bar{\omega},\bar{z})$ defines an equilibrium. Existence and uniqueness of an equilibrium price level are discussed in section 5.

3.3 Comparative statics : effect of a change in the minimum real wage

Let $H(z)$ denote the hessian matrix associated with the production function F, the matrix of the second order derivatives $F_{ij}(z) = \partial^2 F/\partial z_i \partial z_j$ evaluated at z. By the assumption A.1 the matrix $H(z)$ is symmetric, regular and negative-definite for all $z >> 0$. The sign of the cross derivatives indicates the relation which exists between the types of labor : $F_{ij}(z) > 0$ indicates a relation of *complementarity* between type i and type j labor while $F_{ij}(z) < 0$ indicates a relation of *anti-complementarity*.

The equilibrium conditions E.2 define the demand for each type of labor as a function of the real wages, denoted by $z_j = z_j(\omega_1 ... \omega_n)$. By differentiating the first order conditions (1) *for an interior solution* we get the matrix of derivatives of the demand functions with respect to the real wages. It is given by the inverse of the hessian matrix :

$$Z(\omega) = H^{-1}(z(\omega))$$

where $Z(\omega) = [\partial z_i/\partial \omega_j]$. The following proposition covers the effect of an increase in the minimum real wage on the equilibrium real wage-employment pair $(\bar{\omega},\bar{z})$.

Proposition 2 An increase in the minimum real wage ω_o always leads to a decrease in total employment and production. Its effect on the real wage scale and on the distribution of employment depends on the relation between the various types of labor.

To prove that proposition, let us consider a situation where there is full employment in the first k categories. That corresponds to the following system of equations :

(6) $$F_j(L_1 ... L_k, \bar{z}_{k+1} ... \bar{z}_n) = \bar{\omega}_j \quad (j = 1...k)$$

(7) $$F_j(L_1 ... L_k, \bar{z}_{k+1} ... \bar{z}_n) = \omega_o \quad (j = k+1...n)$$

Let us assume that the solution is interior in the sense that $\bar{\omega}_j > \omega_o$ for all $j = 1...k$. Differentiation of the equations (6) and (7) with respect to ω_o leads to the following system of equations :

$$-\frac{d\bar{\omega}_j}{d\omega_o} + \sum_{h=k+1}^{n} F_{jh}(z)\frac{d\bar{z}_h}{d\omega_o} = 0 \quad (j = 1 \ldots k)$$

$$\sum_{h=k+1}^{n} F_{jh}(z)\frac{d\bar{z}_h}{d\omega_o} = 1 \quad (j = k+1 \ldots n)$$

Let us decompose the hessian matrix as follows :

$$H = \begin{bmatrix} H_e & H_k \\ H_k' & H_u \end{bmatrix}$$

where H_e and H_u denote the principal square matrices of order k and n-k respectively. Using that decomposition the system of equations can be written in matrix form :

$$\begin{bmatrix} I & H_k \\ O & H_u \end{bmatrix} \cdot \begin{bmatrix} d\bar{\omega}_e/d\omega_o \\ d\bar{z}_u/d\omega_o \end{bmatrix} = \begin{bmatrix} 0_k \\ 1_{n-k} \end{bmatrix}$$

where 0_k is the 0-vector of dimension k and 1_{n-k} is the 1-vector of dimension n-k. By inverting the above matrix we get the following solution :

$$\frac{d\bar{\omega}_e}{d\omega_o} = (H_k . H_u^{-1}) . 1_{n-k}$$

$$\frac{d\bar{z}_u}{d\omega_o} = H_u^{-1} . 1_{n-k}$$

Let $\bar{E}$ denote total employment. We then have :

$$\frac{d\bar{E}}{d\omega_o} = 1'_{n-k} . H_u^{-1} . 1_{n-k} < 0$$

because H_u is negative definite, as a principal sub-matrix of H. Furthermore production evolves in the same direction as total employment :

$$\frac{d\bar{y}}{d\omega_o} = \sum_{j=k+1}^{n} F_j(\bar{z})\frac{d\bar{z}_j}{d\omega_o} = \omega_o \sum_{j=k+1}^{n} \frac{d\bar{z}_j}{d\omega_o} = \omega_o \frac{d\bar{E}}{d\omega_o} < 0. \qquad \square$$

We observe that the matrix H_e does not enter into account in the evaluation of the effect of change in ω_0 : the effect on real wages and employment does not depend upon the interdependence between categories of labor in which full employment prevails. On the other hand the effect on employment depends only on the interdependence between the categories of labor in which unemployment prevails. In the case where $n = 4$, with full employment in the first two categories, we get :

$$\frac{d\bar{\omega}_1}{d\omega_o} = \frac{1}{\Delta_u}\,[F_{13}(F_{44} - F_{34}) + F_{14}(F_{33} - F_{34})]$$

$$\frac{d\bar{\omega}_2}{d\omega_o} = \frac{1}{\Delta_u}\,[F_{23}(F_{44} - F_{34}) + F_{24}(F_{33} - F_{34})]$$

$$\frac{d\bar{z}_3}{d\omega_o} = \frac{1}{\Delta_u}\,(F_{44} - F_{34}) \qquad \frac{d\bar{z}_4}{d\omega_o} = \frac{1}{\Delta_u}\,(F_{33} - F_{34})$$

$$\frac{d\bar{E}}{d\varpi_0} = \frac{1}{\Delta_u} (F_{33} + F_{44} - 2 F_{34}) < 0$$

where $\Delta_u = F_{33}F_{44} - F_{34}^2$ is the determinant of H_u. It is negative by strict concavity. This example shows that many combinations are actually possible.

4. EQUILIBRIUM WITH A FLEXIBLE QUALIFICATION STRUCTURE

4.1 Definition of an equilibrium

We now introduce the possibility that a worker with a given qualification may actually be qualified for a variety of jobs. That is, for any given qualification j, there is a subset $Q(j) \subseteq \{1...n\}$ of qualifications which are compatible with it. We shall restrict ourself to a *pyramidal* structure in which qualification levels are increasing, starting with the first qualification level, i.e. $Q(j) = \{1...j\}$. In terms of the distribution of employment, it implies the following sequence of constraints :

$$z_n \leq L_n,\ z_{n-1} \leq L_{n-1} + (L_n - z_n),...$$

In general the constraint applying to qualification j is given by:

$$\sum_{k=j}^{n} z_k \leq \sum_{k=j}^{n} L_k \quad (j = 1...n). \tag{8}$$

An equilibrium is similarly defined by a price level $\bar{p}$, a wage vector $\bar{w}$ and an employment vector $\bar{z}$ such that conditions E.1 and E.2 are satisfied, with the conditions E.3 and E.4 being replaced by the following conditions:

E.5 *for all j*, $\bar{w}_j \geq \bar{w}_0$ *and* $\sum_{k=j}^{n} \bar{z}_k \leq \sum_{k=j}^{n} L_k$,

E.6 *for all j*, $\bar{w}_j \geq \bar{w}_{j-1}$ *and* $\sum_{k=j}^{n} \bar{z}_k < \sum_{k=j}^{n} L_k$ *implies* $\bar{w}_j = \bar{w}_{j-1}$.

where $\bar{w}_0 = \bar{p}\varpi_0$. Condition E.5 simply reproduces the restrictions which apply to the real wage rates and to the distribution of employment. Condition E.6 requires the salary scale to be compatible with the qualification structure in the sense that no worker should be *willing* to move to a category in which he or she would be over-qualified. However if there is unemployment in some qualification then the wage rate in that category must equal the wage rate in the next lower qualification. Put differently, a wage difference between two successive categories implies full employment in the most qualified category.

4.2 Characterization and existence of an equilibrium

The dichotomy identified before applies here as well. The existence and uniqueness of an equilibrium pair $(\bar{\omega},\bar{z})$ follows from the following proposition.

Proposition 3 Under the assumptions A.1, there exists *one and only one* pair $(\bar{\omega},\bar{z})$ such that for all j :

(i) $F_j(\bar{z}) \leq \bar{\omega}_j$ *with equality if* $\bar{z}_j > 0$,

(ii) $\bar{\omega}_j \geq \omega_o$ *and* $\sum_{k=j}^{n} \bar{z}_k \leq \sum_{k=j}^{n} L_k$,

(iii) $\bar{\omega}_j \geq \bar{\omega}_{j-1}$ *and* $[\sum_{k=j}^{n} \bar{z}_k < \sum_{k=j}^{n} L_k$ *implies* $\bar{\omega}_j = \bar{\omega}_{j-1}]$.

Condition *(i)* covers condition E.2 and is equivalent to condition *(i)* in proposition 1. Conditions *(ii)* and *(iii)* cover the equilibrium conditions E.5 and E.6. To prove proposition 2, we again consider an optimization program with the same objective function (5), but with a modified employment constraint (8). The arguments used for the case of a rigid qualification structure and based on the Kuhn-Tucker conditions still apply here. The optimization program has a unique solution, denoted by $\bar{z}$, and there exists a vector of non-negative Lagrange multipliers $\lambda = (\lambda_j)$ such that for all j :

$$\lambda_j > 0 \text{ implies } \sum_{k=j}^{n} \bar{z}_k = \sum_{k=j}^{n} L_k$$

and

$$F_j(\bar{z}) \leq \omega_o + \sum_{k=1}^{j} \lambda_k, \text{ with equality whenever } \bar{z}_j > 0.$$

Proposition 3 then follows by defining the real wage rate for type j labor as :

$$\bar{\omega}_j = \omega_o + \sum_{k=1}^{j} \lambda_k \quad (j = 1...n).$$

Indeed $\bar{\omega}_j \geq \bar{\omega}_{j-1}$ because the Lagrange multipliers are all non-negative and a strict inequality is equivalent to $\lambda_j > 0$. □

A possible equilibrium configuration $(\bar{\omega},\bar{z})$ for $n = 5$ is given by :

$$\bar{z}_5 = L_5,\ \bar{z}_4 < L_4,\ \bar{z}_3 + \bar{z}_4 = L_3 + L_4,\ \bar{z}_2 = L_2,\ \bar{z}_1 < L_1,$$

$$\bar{\omega}_5 \geq \bar{\omega}_4 = \bar{\omega}_3 \geq \bar{\omega}_2 \geq \bar{\omega}_1 = \omega_o.$$

Strict inequality between real wages arises in two successive categories when there is a shortage of the most qualified type of labor, a shortage being formally identified by a positive Lagrange multiplier. In the above configuration, only workers in the first category are actually unemployed although workers in the fourth category are actually *under-employed.*

It is interesting to notice that the budget deficit associated with an equilibrium pair $(\bar{\omega},\bar{z})$ is again given by equation (4). Indeed if an equality arises in the sequence of constraints (8) for some qualification then it means that the workers whose qualification is equal to or higher than that qualification are actually all employed although possibly under-employed. As a consequence unemployment compensations if any are computed on the basis of the minimum wage, that is unemployed workers all receive a real compensation equal to $\gamma \omega_0$. Equilibrium prices are defined as for the case of a rigid qualification structure on the basis of the real wage-employment pair $(\bar{\omega},\bar{z})$.

4.3 Comparing the qualification structures

It would be interesting to evaluate how employment, production and deficit change when moving from the rigid qualification structure to a flexible one by allowing qualified workers to accept less qualified jobs. There is however no general answer to that question. The redistribution of employment between the various types of labor may well end up with a lower employment level i.e. a higher unemployment rate. Furthermore a lower employment level may well be compatible with a higher production level. All we know is that the resulting real profit computed at the minimum real wage, as defined in (5), cannot be lower because the employment constraints define a larger set under a flexible qualification structure.

In the case where there are only two types of labor and some low skill workers are initially unemployed, the distribution of employment remains unchanged. In the opposite situation where all low skill workers are initially employed, the number of low skill jobs increases although total employment may not be larger. In this situation full employment results if the *unconstrained* maximum of the real profits implies an overall excess demand for labor.

To answer the above question it would be necessary to specify the technological interrelationships between the various types of labor as measured by the second order cross derivatives of the production function. Moreover the qualification structure should be explicitly linked to the productivities and skills, and to the relative scarcity of the various types of labor.

5. EXISTENCE OF AN EQUILIBRIUM

Until now the existence of an equilibrium has been proven only in real terms. It remains to find conditions under which there exists an equilibrium price level.

5.1 Existence of an equilibrium price level

The following arguments apply independently of the qualification structure and start from an equilibrium pair $(\bar{\omega},\bar{z})$. To prove existence of an equilibrium price level, we consider the excess demand function $f: \mathbb{R}_+ \rightarrow \mathbb{R}$ defined by:

$$f(p) = C(p,p\bar{\omega},\bar{z}) - \bar{y}$$

This function is continuous at positive prices and, by assumption A.4, $f(p) > 0$ for p high enough. On the other hand, by assumption A.2, we have

$$f(p) < \frac{M_o}{p} + (1-t)\,[\Sigma_j\bar{\omega}_j\bar{z}_j + \gamma\Sigma_j\bar{\omega}_j(L_j-\bar{z}_j)] + (1-\theta)\,[\bar{y} - \Sigma_j\bar{\omega}_j\bar{z}_j] - \bar{y}$$

Rearranging the terms at the right hand side of the above expression, we get

$$f(p) < \frac{M_o}{p} + \bar{d} \quad \text{for all } p > 0$$

where $\bar{d}$ is the real budget deficit associated to $(\bar{\omega},\bar{z})$ as defined by equation (4), with $\bar{d} \leq 0$. Hence $f(p) < 0$ for p large enough. By continuity there exists $\bar{p} > 0$ such that $f(\bar{p}) = 0$. Under assumption A.3, f is a decreasing function and consequently $\bar{p}$ is unique. This establishes the following proposition:

Proposition 4 Under the assumptions A.1 to A.4, there exists *one and only one* equilibrium price level if there is *no real deficit.*

Using the budget constraint (2) the stock of money at the end of the period is given by: $M_1 = M_o + \bar{d}$. This shows that the absence of deficit in real terms is equivalent to the absence of money creation. It is a *sufficient* condition under which there exists an equilibrium. When there is a deficit, inflation *may not* lead the economy to an equilibrium. That condition places joint restrictions on the level of the minimum of subsistence and on the fiscal parameters t, θ and γ.

5.2 Sustainability of the minimum income rule

The question concerns the capacity of the economy to finance the unemployment compensations from income taxes, that is the existence of tax rates t and θ such that there is no deficit (nor surplus) given the minimum of subsistence.

The problem is that unemployment implies a lower production and this reduces the redistributive capacity of the economy. Moreover the tax on labor income introduced to finance the unemployment compensations raises the minimum real wages and thereby lowers further the level of employment. These two effects contribute to create conditions under which the current level of activity may not be sufficient to cover the needs of the population although it is has been assumed to be potentially feasible.

Alternative institutional arrangements like employment subsidies perform better in this framework. The idea is that firms receive subsidies to motivate them to employ all workers. More precisely, if $F_j(L) < e/(1-t)$ for labor of type j, firms receive a subsidy equal to the difference : workers in category j cost their marginal productivities but receive a *net* real wage equal to the minimum of subsistence. In this setting there is full employment and an equilibrium price system is defined by $\bar{p} > 0$ such that:

$$C(\bar{p},\bar{w},L) = F(L)$$

where the nominal wages $\bar{w}$ are given by:

$$\bar{w}_j = \bar{p}\ \mathrm{Max}[\frac{e}{1-t}, F_j(L)] \quad (j = 1...n).$$

The associated real deficit is given by:

$$\bar{d} = \Sigma_j[\bar{\omega}_j - F_j(L)]L_j - t\, \Sigma_j\bar{\omega}_jL_j - \theta\, [F(L) - \Sigma_jF_j(L)L_j] \tag{9}$$

where the $\bar{\omega}_j$'s are the equilibrium real wage rates defined by $\bar{\omega}_j = \bar{w}_j/\bar{p}$. The equilibrium price level is then solution of the $C(p,p\bar{\omega},L) = F(L)$. Proposition 3 carries over in the sense that an equilibrium exists and is unique whenever there is no (real) deficit. The following proposition establishes that it is actually possible to cover the subsidies from taxes :

Proposition 5 Under the assumptions A.1, A.5 and A.6, there exist θ and t such that $\bar{d} \leq 0$.

Proof. From equation (9), the real deficit can be written as a function of t and θ :

$$d(t,\theta) = \Sigma_j\ \mathrm{Max}[e, (1-t)F_j(L)]\, L_j + (\theta-1)\, \Sigma_jF_j(L)L_j - \theta\, F(L)$$

Let $\bar{t}$ denote the tax rate at which *all* sectors receive subsidies. It is defined by the equation $e = (1-\bar{t})\ \mathrm{Max}_jF_j(L)$, where $\bar{t} \in [0,1]$ by the assumption A.5. At $t = \bar{t}$ we have :

$$d(\bar{t},\theta) = e\, \Sigma_jL_j + (\theta-1)\, \Sigma_jF_j(L)L_j - \theta\, F(L).$$

The equation $d(\bar{t},x) = 0$ has a unique solution in x which is given by :

$$\bar{x} = \frac{\Sigma_j (r - F_j(L))L_j}{F(L) - \Sigma_j F_j(L) L_j}$$

The denominator is positive by the assumption of strict concavity and $\bar{x} \le 1$ by the assumption A.6. At $\bar{\theta} = \mathrm{Max}(0,\bar{x})$. we have $d(\bar{t},\bar{\theta}) \le 0$. □

As a consequence, there are tax rates such that there exists a (unique) equilibrium with full employment enforced via subsidies.

6. CONCLUDING REMARKS

In this paper, we have investigated the possible consequences of the implementation of different *social contracts* in a framework which allows for an heterogeneous labor force. The pure "walrasian contract" may lead to the (involuntary) decrease of the population, even if it does not loose its basic welfare properties. The "unemployment compensation cum minimum wage contract" - which is almost universal in industrial countries - may be unsustainable as the occurrence of unemployment reduces the redistributive capacity of the economy. The spirit of this kind of of social arrangement is to provide everyone with a minimum subsistence level, even if some remain unemployed. Hence the idea of a third kind of social contract which could provide (theoretically) everyone with a minimum subsistence level *and* a job. We have seen that this may be the case of an employment subsidy scheme which appears more likely to be sustainable than the unemployment compensation scheme although we do not investigate the practical difficulties of implementing it.

The salary scale which comes out of our modeling is related to the qualification structure. The term "qualification" is here defined in terms of the ability of workers to perform particular tasks ; the larger the set of jobs a given worker can access to, the more qualified he or she is. Qualification is therefore not directly related to productivity although the corresponding wage level is. While we have restricted ourself to the case of a pyramidal qualification structure, it is straightforward to extend the analysis to the case of more complex structures, involving for instance separate qualification trees.

Our analysis has been greatly simplified by the dichotomy which results from the equilibrium definition which is a consequence of several assumptions. In particular we have worked with an aggregate consumption good. Allowing for several consumption goods requires a concept of equilibrium *relative to a price system* and a price index to measure real wages. The assumption of an inelastic labor supply is a further assumption which allows for a simple characterization of an equilibrium in real terms. That characterization is lost but the dichotomy and the existence result remain valid when the labor supplies depend on *real* wages. In this case the social product may be lower as the redistributive scheme may have adverse effects on the supply of labour of the more qualified workers and thus be less effective.

Our analysis is short term in nature. Allowing for existence of capital goods would introduce further complications but would be worthwhile investigating. It would indeed allow for the study of the interrelationship between the real interest rate, the real price of capital and the qualification structure. It would also allow for the study of the capacitive of the economy to finance the unemployment compensations or the employment subsidies out of a richer tax structure. These limitations pave the way for further research.

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