NON-OBSERVABLE NOISES AS A POSSIBLE CAUSE OF CONDITIONAL HETEROSCEDASTICITY: *THE CASE OF INTRADAY EXCHANGE RATES*

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ABSTRACT:
In this paper, a model of the dynamics of intradaily exchange rates is presented. Banks are risk neutral; they are either price-taker (when not quoting) or price maker (when quoting). Each bank quotes in its turn. The current Over-The-Counter (OTC) exchange rate is the quote of the quoting bank i.e. the expectation of the exchange rate at the forthcoming fixing conditional to its public and private information. Commercial traders' orders are exogenous and portofolio theory is used to model speculators' behaviour. Commercial orders are noisy; the noises are assumed to be independent, zero-mean and homoscedastic. Two polar cases are considered: (i) If each bank is able to observe the noises relative to the orders of its own clients, then the OTC exchange rate is shown to obey a random walk with a constant conditional variance. (ii) If each bank is not able to observe the noises relative to the orders of its own clients, which is the case in practice, it will look for a good predictor of the exchange rate at the forthcoming fixing which may be a linear combination of the observed variables that span its information structure. The OTC exchange rate is no more a random walk; its conditional variance is the sum of a quadratic form of past values of the exchange rate (this is the conditional heteroscedasticity effect) and of an independent random variable (stochastic volatility). Conditional heteroscedasticity appears to be closely related to non-observability of noises.

KEYWORDS: ARCH models, conditional variance, efficiency, exchange-rate, heteroscedasticity, intraday, rational expectations, stochastic volatility models.

JEL CLASSIFICATION: F3, C6, G0.

1. INTRODUCTION

Empirical studies[3] report that exchange rates changes exhibit conditional heteroscedasticity and autocorrelation. Such a phenomenon was first pointed out by Mandelbrot in his 1963 seminal paper. Since the early eighties, parametric statistical models, e. g. AutoRegressive Conditional Heteroscedasticity (ARCH) models have been developped.[4] In this class of models, volatility is endogenously determined by an autoregressive process. Continuous-time processes have also been considered.[5] Whatever the point of view, these approaches are but empirical.

This paper is an attempt in suggesting a theoretical explanation to the conditional heteroscedasticity of time series of financial asset prices. The intuition of the paper can be first presented by using a very simple model, somewhat in the spirit of Muth's (1961) one. Consider a walrasian market on which a good is traded. The demand for this good is supposed to depend on two elements: its price p_t and a random noise u_t. For the sake of simplicity, we shall assume a linear demand function with zero-mean and i.i.d. noises; hence:

$$q_t = -a\ p_t + u_t\ , \qquad E[u_t u_s] = \delta_{ts}\sigma_u^2 \qquad E[u_t] = 0\,.$$

On the supply side, the « representative producer » is supposed to face a deterministic quadratic cost function:

$$C(q_t) = \alpha\ q_t^2\ /\ 2 + \beta\ q_t + \gamma\ ,$$

and its profit reads:

$$\Pi_t = p_t\ q_t - C(q_t)\,.$$

The representative producer maximises his expected profit; hence:

$$\frac{d}{dq_t} E[\Pi_t | \mathrm{I}_{t-1}] = E[p_t | \mathrm{I}_{t-1}] - \alpha\ q_t - \beta = 0\,.$$

The level of output depends of the expectation, by the producer, of what the price will be:

$$q_t = (E[p_t | \mathrm{I}_{t-1}] - \beta)\ /\ \alpha\ .$$

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3 For a recent survey of these works see e.g. Dacorogna et alii (1993). The higher the time scale the less heteroscedastic and the less autocorrelated the data seem to be

4 For a recent survey see e.g. Bollerslev, Chou and Kroner (1992).

5 For a recent survey see e.g. Duffie (1989, 1992). The equivalence between GARCH (1,1) models and diffusion processes has been shown, under some restrictive assumptions, by Nelson (1990).

The market equilibrium equation reads:

$$(E[p_t|\mathrm{I}_{t-1}]-\beta)/\alpha=-a\ p_t+u_t.$$

Taking conditional expectations on both sides of this last equation leads to the following equation determining the value of the price expectation $E[p_t|\mathrm{I}_{t-1}]$:

$$E[p_t|\mathrm{I}_{t-1}]=(\beta+\alpha\ E[u_t|\mathrm{I}_{t-1}])/(1+\alpha\ a).$$

Two polar cases must now be distinguished, according to the definition of the information set of the representative producer:

(i) The producer *observes* the noises u, i.e. he *knows*, at time t,[6] the values of the sequence of past noises (u_{t-k}, $k=1, ..., K$). Since he also knows the sequences of past prices and quantities, then he knows the true value of a. Mathematically, we shall say that I_{t-1} is the tribe generated by the set of three lagged variables :

$$\mathrm{I}_{t-1}=\sigma(p_{t-k}, q_{t-k}, u_{t-k};\ k=1, ..., K).$$

The price p_t is the sum of u_t and of a deterministic constant: the p's are i.i.d. and zero-mean and, therefore homoscedastic. More precisely we get:

$$E[p_t|\mathrm{I}_{t-1}]=\beta/(1+a\alpha),$$

$$Var[p_t|\mathrm{I}_{t-1}]=\sigma^2(p_t)=\sigma_u^2/a^2.$$

(ii) The producer *cannot observe* the noises u: I_{t-1} is now the tribe generated by the set of two lagged variables :

$$\mathrm{I}_{t-1}=\sigma(p_{t-k}, q_{t-k};\ k=1 ..., K).$$

Before he decides what will be the value of his ouput, he must infer the demand curve. To do so, he will look for a good predictor $\hat{p}_t$ of p_t, which will depend on past values of p's and q's. He can think, for instance, of a regression of p on q (for the K available observations). We thus write:

$$\hat{p}_t=\hat{f}_t(p_{t-1}, q_{t-1}, ..., p_{t-K}, q_{t-K})\ q_t.$$

He can now choose the level of output, maximizing its expected profit; hence:

$$q_t=(\hat{p}_t-\beta)/\alpha$$

and the equilibrium condition reads:

$$(\hat{p}_t-\beta)/\alpha=-ap_t+u_t$$

or, finally:

6 Just before he decides about the volume of output.

$$p_t = (\beta - \hat{p}_t) / a\alpha + u_t / a .$$

The price p_t is no more stationary since we get:

$$E\left[p_t \middle| \mathrm{I}_{t-1}\right] = (\beta - \hat{p}_t) / a\alpha ,$$

$$Var\left[p_t \middle| \mathrm{I}_{t-1}\right] = (Var\left[\hat{p}_t \middle| \mathrm{I}_{t-1}\right] / \alpha^2 + \sigma_u^2) / a^2 .$$

Two questions now arise: (i) How important should be this phenomenon? (ii) Can we generalize this result to financial asset prices and more precisely to exchange rates?

It is obvious that if our representative producer keeps in memory the sequence of past prices and quantities, his estimation will, *ceteris paribus*, become more and more accurate and $\hat{p}_t$ will tend asymptotically to $\beta / (1 + a\alpha)$. Heteroscedasticity is counter balanced by learning and the usual linear rational expectations model can then be viewed as a good approximation of reality. This is no more the case as far as private information is taken into account since it will remain non-observable by agents who do not get it. This will be the case, even though information is partly conveyed from one informed agent to an uninformed one through prices or volumes.

To illustrate the last proposition, let us turn to exchange rates. It is convenient to reason as if two markets would exist: an over-the-counter (OTC) market, which is opened almost continuously, and an auction market which is supposed to take place at the end of each day. All orders are then centralised. Banks, in number B, intervene in the OTC market. Trading arrangements are such that any bank must be a price maker P times a day and a price taker $(B\text{-}1)\,P$ times. The current exchange rate is thus the quote of the price maker, e. g. the quoting bank. Since banks are supposed to be, on the short period, risk-neutral, transactions are supposed to be bounded and the quote of the quoting bank, i.e. the current exchange rate e_t, must be equal to its expected value[7] at the end of the day; hence:

$$E\left[e_D \middle| \mathrm{I}_t\right] = e_t$$

This will be the case if common knowledge prevails. If not, it will equalize the expectation of e_D conditional to the quoting bank's information $\mathrm{I}^i{}_t$:

$$E\left[e_D \middle| \mathrm{I}^i{}_t\right] = e_t$$

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7 We do not care for discounting.

I^i_t will include bank i's private information and public information (e.g. the sequence of past exchange rates since the opening of the OTC market) available at time t. Private information can be thought as the set of the values of bank i's clients' orders. Two cases must be distinguished:

(i) Each bank *observes* its own noises, i.e. it is able to know what the value of its clients' orders should exactly be in the absence of noises, and, consequently, the difference between the « theoretical » values and the corresponding effective values. It can thus estimate $E\left[e_D \middle| I^i_t\right]$ accurately.

(ii) Each bank *does not observe* its own noises. In that case, each bank can only look for a good approximation of $E\left[e_D \middle| I^i_t\right]$. Conditional heteroscedasticity will be caused by the variance of the estimators of the coefficients of the predictor $\hat{e}^i_t$ of $E\left[e_D \middle| I^i_t\right]$.

The link between the two markets is due to the assumption that every bank clears its position at the end of each day. Using the framework of Broze, Gourieroux and Szafarz (1989), it can be shown that the equilibrium condition for the auction market will express as a stochastic difference equation.

To sum up, we have chosen the case of exchange rates to develop the rigourous calculations necessary for modelizing how imperfect observability of noises make conditional heteroscedasticity of prices to rise. For the sake of simplicity the « fundamental » value of the exchange rate will be viewed as the value allowing for the equilibrium of the market, once banks have cleared their position and will be identified to the old fixing procedure. Finally, our *conjecture* is that the conditional heteroscedasticity depends crucially on the accuracy with which each trader in the market -all are rational- observes his private information. Two related questions will have to be examined in detail :

(i) How accurate is the trader's observation of his private information? If the private information is stochastic, as it is assumed in the paper, then does each trader observe, or not, the noises relative his private information?

(ii) How private information is conveyed to the market?

In financial markets, prices and volumes are generally known by all the traders. Nevertheless, to simplify the investigation of the existence of conditional heteroscedasticity of time series of financial asset prices, the informative role of volumes will be neglected; private information will be here supposed to be conveyed to the market only through prices. Such an simplification is not drastic, since volumes are not generally public information on the Over The Counter (OTC) foreign exchange market. The functioning of this market will even be more simplified by considering that: (i) when it is quoting, each bank quotes only one price instead of two (the bid and ask prices) and faces an infinite elasticity to the other banks demand for currency (this avoids to make an expectation of the demand), (ii) the volume of its correspondents' orders is not taken into account in the bank's information structure. With this simplification of the functioning of the market, the quotes do not contain any information relative to volumes. Such a choice is justified in the next section (see §2.A). Moreover, knowing its private information is not complete, each bank is aware of the other banks' private information which are reflected by their quotes. So quotes convey some information from one bank to another. Note that this phenomenon is different from mimetic contagion (e.g. Kirman (1993) Orléan (1992), Orléan and Robin (1991) and Topol (1991)) from "sun-spots" literature (Azariadis (1981), Azariadis and Guesnerie (1982)) or "fads" models (for a survey see e.g. West (1988)).

The paper is organised as follows: the functioning of the OTC market is presented in Section 2; the bank's optimal behaviour is described in Section 3; in Section 4, the dynamics of the OTC exchange rate and a theoretical explanation of conditional heteroscedasticity are presented; the conclusion is then given.

2. THE FUNCTIONING OF THE OTC FOREX MARKET

For the sake of simplicity and without loss of generality the OTC forex market is viewed as a *bilateral* forex market (two countries and one foreign exchange rate) where two currencies, the domestic one, namely *d*, and the foreign one, namely *f*, are distinguished. Note that the foreign currency *f* is quoted in the domestic currency *d*, i.e. one unit of $f = e$ unit(s) of *d*.

A. Trading arrangements

Domestic banks are in number *BD* and foreign banks are in number *BF*; the total number of banks is $B = BD + BF$. Banks are labelled from 1 to *B*. Each bank intervenes whatever its nationality either:

(i) as a "*price maker*"; it is then quoting and has to make the counterpart of all orders passed by other banks;

(ii) or as a "*price taker*"; it then trades -for itself and/or its clients- the amount of currency it desires with the bank which is quoting.

We assume that this amount is bounded, for instance in terms of foreign currency. The corresponding bound is $q^{f}*$. Such an assumption is made for two reasons: (i) Trading arrangements prevailing in the OTC forex market are usually that quantities are normalised to one million USD. (ii) We shall later assume risk neutrality for banks which implies bounded transactions. Finally note that a bank is obliged to quote otherwise it will finally be put off the market.

As a rational agent whose optimal behaviour is presented in the next section, each bank has a horizon at which it maximises its expected wealth. It is also the date at which it is supposed to clear its position in its risky currency (i. e. the foreign currency for a domestic bank and the domestic currency for a foreign bank). One point of view would be to consider that bank's horizons are heterogeneous. This model of the OTC forex market would be rather complicated. For the sake of simplicity, a common horizon is assumed, e. g. it is identified to the fixing.[8]

The opening period of the OTC market -the intraday period- is divided into P sub-periods in which each bank quotes only once; for the sake of simplicity, the following assumption is made:

ASSUMPTION 1: *The opening period of the OTC market is divided into an exogenous constant number of P sub-periods and the banks are quoting sequentially in each sub-period.*

This assumption could be relaxed without harm to the results. Indeed, the times at which commercial and speculative orders are given to the bank and at which transactions with banks are made could be modelled by a Poisson process. Moreover, the function $t \mapsto \theta(t)$ could be a random drawing in a uniform discrete distribution defined over the set of digits from 1 to B instead of being a deterministic injection. Finally the parameter P could be endogenized. Since B banks operate in the OTC market, there are $B\ P$ quoting times between two consecutive fixings (D-1 and D); D labels the current day. During each

[8] The common horizon is called the *fixing* throughout the paper.

OTC "day", denoted[9] $\{(D-1,\ D)\}$ for the D^{th} day, each bank quotes P times, hence t varies from 1 to BP within an opening of the OTC market.

At time t, the quoting bank is defined by the value of the function $\theta : \mathbf{N} \to [1,\ B]\cap\mathbf{N}$ which determines its label:

$$\theta(t) = i. \tag{1a}$$

The current time t reads:

$$t = (p-1)B + i, \qquad \text{with } i = 1 \text{ to } B \text{ and } p = 1 \text{ to } P$$

or

$$\theta(t) = t \quad \text{modulo } B.$$

The quote, $e_t^{\theta(t)}$, of the quoting bank, $i = \theta(t)$, is but the OTC market exchange rate e_t prevailing at that time, hence:

$$e_t = e_t^{\theta(t)}. \tag{1b}$$

The labelling of variables is, thus, as shown on the following figure, for the exchange rate:

$$\text{---}e_{D-1}\text{--}]\text{---------------}e_t\text{-------------------}[\text{--}e_D\text{--}$$

*Last Fixing Time*____________ *Current Time*____________ *Next Fixing Time*

Moreover, we assume that the bank which is quoting is facing the demand from its correspondents and we provisionally make the following assumption:

ASSUMPTION 2: *Each bank has exactly* $(B-1)$ *correspondents.*

This functioning of the OTC forex market which has been presented has introduced:

(i) A forex market which consists of an OTC market crossed with an auction market. This kind of "crossed-market" microstructure for a speculative market where agents are either price maker or a price taker is quite unusual.

(ii) Two time scales: the shorter one characterises the OTC period and the longer one the interval between two fixing times. To these time scales correspond two different behaviours: risk neutrality of banks trading in the OTC market and risk aversion of speculators.

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9 The short hand notation $\{(D-1,\ D)\}$, instead of $(D-1,\ D)\cap\mathbf{N}$, is chosen for the sake of simplicity.

Commercial orders are financial counterparts of exports and imports. Without loss of generality, we assume that exports (imports) are denominated in domestic (foreign) currency.[10] As the OTC time scale is very small, it is rather sensible to think of exports and imports being exogenous. Of course their prices and volumes and hence their values vary with, among other variables, the exchange rate, but it is well known (see e.g. Magee (1973)) that they react slowly to fluctuations of exchange rates. However, we allow for noisy commercial orders; hence monthly imports or exports read:

$$\tilde{M}_M^f = M_M^f + \tilde{v}_M^m \tag{2a}$$

$$\tilde{X}_M^d = X_M^d + \tilde{v}_M^x \tag{2b}$$

where M_M^f and X_M^d are the deterministic components respectively of imports and of exports, $\tilde{v}_M^m$ and $\tilde{v}_M^x$ are the corresponding noises and the label M is indicating the month.

The deterministic components correspond to the planned transactions and their realisations depend on noises. Daily commercial orders are modelled in a similar manner; hence:

$$\tilde{M}_D^f = \lambda_D^f M_M^f + \tilde{v}_D^m \tag{3a}$$

$$\tilde{X}_D^d = \lambda_D^d X_M^d + \tilde{v}_D^x \tag{3b}$$

where λ_D^f (λ_D^d) and $\tilde{v}_D^m$ ($\tilde{v}_D^x$) are respectively the proportion of the deterministic component of the monthly imports (exports) and the stochastic component of imports (exports) on D-day, namely the daily noises. Assuming 30 days per month, the relationship between the noises reads:

$$\sum_{D=1}^{30} \tilde{v}_D^f = \tilde{v}_M^m \text{ and } \sum_{D=1}^{30} \tilde{v}_D^d = \tilde{v}_M^x \tag{3c}$$

The amount of the daily commercial orders reaching bank i at time t reads:

$$\tilde{M}_t^{if} = \lambda_t^{if} \lambda_D^f M_M^f + \tilde{v}_t^{if} \tag{4a}$$

$$\tilde{X}_t^{id} = \lambda_t^{id} \lambda_D^d X_M^d + \tilde{v}_t^{id} \tag{4b}$$

where λ_t^{if} (λ_t^{id}) and $\tilde{v}_t^{if}$ ($\tilde{v}_t^{id}$) are respectively the proportion of the deterministic component of the daily imports (exports), i. e. the market share of bank i at time t, and the stochastic component of imports (exports) reaching bank i at time t.

The λ's and the ν's verify the following constraints:

$$\sum_{i=1}^{B} \sum_{t=1}^{BP} \tilde{v}_t^{if} = \tilde{v}_D^m \text{ and } \sum_{i=1}^{B} \sum_{t=1}^{BP} \tilde{v}_t^{id} = \tilde{v}_D^x \tag{5a}$$

$$\sum_{i=1}^{B} \sum_{t=1}^{BP} \lambda_t^{id} = 1 \text{ and } \sum_{i=1}^{B} \sum_{t=1}^{BP} \lambda_t^{if} = 1 \tag{5b}$$

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10 Other assumptions about the currencies in which commercial orders are denominated could be made without modifying our results.

Note that:

$$\tilde{v}_D^f \neq \lambda_D^f \, \tilde{v}_M^m, \quad \tilde{v}_D^d \neq \lambda_D^f \, \tilde{v}_M^x$$
$$\tilde{v}_t^{if} \neq \lambda_t^{if} \, \tilde{v}_D^m, \quad \tilde{v}_t^{id} \neq \lambda_t^{if} \, \tilde{v}_D^x \qquad (5c)$$

since daily imports (exports) should not be perfectly correlated with the monthly ones nor with the orders reaching one bank at time t. The four inequalities (5c) are justified to consider the general case of imports and exports reaching bank i at time t which are not 'representative' of the whole market.

The content of equations (2a) to (5c) are summarised in the following assumption:

ASSUMPTION 3: (i) *The commercial orders reach the banks, day after day at the current time t during the opening of the OTC market according to the set of equations (4) and (5).*

(ii) *The noises of the imports and the exports, i. e.* $\tilde{v}_t^{if}$ *and* $\tilde{v}_t^{id}$ $t = 1$ to $B^2 \ P$, $i = 1$ to B, *are assumed to be zero-mean independent and homoscedastic*:

$$E\left[\tilde{v}_t^{if}\right] = E\left[\tilde{v}_t^{id}\right] = 0 \quad \Rightarrow \quad E\left[\tilde{v}_D^x\right] = E\left[\tilde{v}_D^m\right] = 0$$
$$E\left[\tilde{v}_t^{if} \, \tilde{v}_u^{jf}\right] = \delta_{tu} \, \delta_{ij} (\sigma_v^f)^2$$
$$E\left[\tilde{v}_t^{id} \, \tilde{v}_u^{jd}\right] = \delta_{tu} \, \delta_{ij} (\sigma_v^d)^2 \qquad (6)$$
$$E\left[\tilde{v}_t^{if} \, \tilde{v}_u^{jd}\right] = 0$$

where δ_{ij} is the Kronecker delta and $(\sigma_v^f)^2$ $\left((\sigma_v^d)^2\right)$ is the variance of the noise $\tilde{v}_t^{if}$ of the imports ($\tilde{v}_t^{id}$ of the exports). Three remarks can be made:

(i) The independence assumption is not necessary but is made to simplify the calculation. Consequently, if under this simple assumption, the OTC exchange rate appears to be conditionally heteroscedastic, it enforces the theoretical explanation.

(ii) The assumption of a uniform desaggregation of the deterministic parts of the orders can be made for the sake of simplicity, i. e. the coefficients λ's can be equal to $1/B^2 \ P$.

(iii)The noises of each bank's commercial orders, i. e. $\left(\tilde{v}_t^{id}, \tilde{v}_t^{if}, t = 1 \text{ to } B^2 \ P, i = 1 \text{ to } B\right)$ make these orders a piece of private information.

The daily commercial balance may be calculated by aggregating commercial orders over the banks and time. It is convenient to use a linearisation -a first order Taylor expansion-around the average value e_{M-1} of the exchange rate during the previous month:

ASSUMPTION 4: *The daily trade balance* $\left(\tilde{X}_D^f - \tilde{M}_D^f\right)$ *is a linear function of the exchange rate:* $\tilde{X}_D^f - \tilde{M}_D^f \approx \tilde{u}_D - \delta_D \tilde{e}_D$ (7)

where $\tilde{u}_D$ and δ_D respectively read:

$$\tilde{u}_D = u^*{}_D + \tilde{v}_D, \tag{8a}$$

$$u^*{}_D = \lambda_D \left(2\, X_M^d / e_{M-1}\right), \tag{8b}$$

$$\tilde{v}_D = \left(\tilde{v}_D^x / e_{M-1}\right) - \tilde{v}_D^m \tag{8c}$$

$$\delta_D = \lambda_D\, X_M^d / e_{M-1}^2 \tag{8d}$$

Note that the daily trade balance is expressed in terms of (i) monthly values of imports and exports; (ii) past values of monthly exchange rates and (iii) stochastic processes $\left(\tilde{v}_D^x, \tilde{v}_D^m\right)$ which are the sum of the "instantaneous" noises, i. e. $\left(\tilde{v}_t^{id} \tilde{v}_t^{if}, t = 1 \text{ to } B^2\ P, i = 1 \text{ to } B\right)$. The expression of the daily noises, equation (8c), is the result of a linearisation in which the term like $\tilde{v}_D^x \left(\dfrac{e_{M-1} - e_D}{e_{M-1}\, e_D}\right)$ is neglected.

B. Speculators

Both domestic and foreign speculators intervene in the market. The modelling of the speculator's behaviour is in the line of the Rational Expectations Equilibrium and Noisy Rational Expectations Equilibrium models (Grossman, 1976, Grossman 1981, Bray, 1985, Admati, 1987 and Admati and Pfeiderer, 1988). The behaviour of domestic and foreign speculators is presented in appendix A; the main results of which are recalled.

ASSUMPTION 5: *Speculative orders are assumed to be denominated only in foreign currency. Each speculator determines his demand for foreign currency just after the fixing at* $D-1$ *and before the opening of the next OTC market, his/her total amount of orders to be passed and executed during period* $\{(D-1,\ D)\}$. *His/her decision is conditional upon his available information at* $D-1$.

Unlike assumption 4, assumption 5 is rather crude since one could expect the speculator to continuously revise his/her expectations and take his/her decision with respect to his/her information at time *t*; he/she would then behave as a commercial bank. This assumption is thus made for the sake of simplicity; it can be relaxed by assuming he/she revises his/her expectations by using the same method than the banks (see section 3).

ASSUMPTION 6: *Speculators are risk averse; they have a CARA utility function. Foreign speculator's risk aversion is constant in domestic currency. Transaction costs are neglected. Common knowledge among speculators and normality*[11] *of the exchange rate at the fixing, i. e.* $\tilde{e}_D$, *are assumed. Speculator's information structure consists of the set of past values of the exchange rate at the fixing (the tribe generated by); it reads:*

$$\mathrm{FI}_{D-1} = \sigma\left(\tilde{e}_{D-K}, 1 \le K \le D-1\right).$$

Since the fixing exchange rate is a linear function of the daily noises of commercial orders and that a rational expectations model is assumed to describe the speculators' behaviour, the total demand for foreign currency during D day reads (appendix A, equation (A.15)):

$$S\tilde{P}E *_D^f = \frac{g_D^f}{b\ \sigma_D^2\ R_{D-1}^f} NS_{D-1} - \frac{g_{D-1}^f}{b\ \sigma_{D-1}^2\ R_{D-2}^f} NS_{D-2}, \tag{9}$$

with:

$$\tilde{g}_D^f = E\left[\tilde{e}_D \middle| \mathrm{FI}_{D-1}\right] - \rho_{D-1}\, e_{D-1}, \tag{10}$$

$$\sigma_D^2 = Var\left(\tilde{e}_D | \mathrm{FI}_{D-1}\right), \tag{11}$$

$$\rho_D = R_D^d / R_D^f = \left(1 + r_D^d\right) / \left(1 + r_D^f\right) \approx 1 + \left(r_D^d - r_D^f\right) \tag{12}$$

where $\tilde{e}_D$ is the exchange rate at the fixing time D, r_D^d (r_D^f) is the domestic (foreign) interest rate -on D day and on a daily basis-, NS_{D-1} is the total number of speculators whose set is denominated ST_{D-1} and b is the harmonic mean of the speculators' risk aversion $\left(NS_{D-1} / b = \sum_{k \in ST_{D-1}} 1/b^k \right)$. Equation (9) corresponds to the fact that the amount invested by a domestic speculator in foreign currency is proportional to the unitary expected gain and is in inverse proportion to its relative risk aversion and the conditional variance of the exchange rate at the forthcoming fixing.

Under the assumption of a constant number of speculators and speculator's risk aversion, zero interest rate differential, the demand for foreign currency speculation reads (see appendix C, equation C.32):

$$\beta\ SP\tilde{E} *_D^f = \mu_D - \chi\ \mu_{D-1} + \rho\ \mu_{D-2} + \left(\psi_1 - \rho\ \psi_0\right) \tilde{v}_{D-1} + \sum_{K=0}^{D-1} \left(\psi_{K+2} - \chi\ \psi_{K+1} + \rho\ \psi_K\right) \tilde{v}_{D-1-K}$$

where the parameters and variables are determined by the equilibrium condition of the fixing.

[11] More generally, elliptical distributions could be assumed.

Finally, the speculative orders to bank i at time t is given by the following equation:

$$S\tilde{P}E_t^{if} = \lambda'^{if}_t \; S\tilde{P}E*^f_D \tag{13}$$

where the total expected speculative demand for foreign currency $S\tilde{P}E*^f_D$, is given by equation (9) and the coefficients λ verify the following constraints:

$$\sum_{i=1}^{B}\sum_{t=1}^{BP} \lambda'^{if}_t = 1. \tag{14}$$

The total of orders to be executed at time t by bank i is defined as the number of units of foreign currency against the domestic one that bank i trades with its clients, it reads:

$$T\tilde{O}I_t^{if} = \tilde{X}_t^{id} / \tilde{e}_t - \left(\tilde{M}_t^{if} + S\tilde{P}E_t^{if} \right) \tag{15}$$

where $\tilde{e}_t$ is the exchange rate at time t. At he fixing at time D, commercial traders and speculators can no more pass any orders. All the orders, as previously discussed, are assumed to be independent of the current exchange rate $\tilde{e}_t$ in the OTC market.

3. INDIVIDUAL BANK'S OPTIMAL BEHAVIOUR

We now turn to the individual bank's bahaviour: it depends on its information structure and on its risk aversion.

A. *Individual bank's information structure*

According to the functioning of the OTC forex market and the specification of the commercial and speculative orders (see section 2), at any time during the opening of the OTC market, several sources of information is available to the banks.

ASSUMPTION 7: *The sources of information of each bank are the following ones:*

1) *The* public information:

(i) *the set of the past values of the fixing exchange rate, which is a* linear *function of the daily noises;*

(ii) *the set of the previous quotes on the current OTC market;*

2) *The* private information *is*:

(i) *the quantities of currency asked to the quoting bank by the others; they are not here taken into account to simplify the number of channels which convey information to the market;*

(ii) *the total orders which are the sum of the commercial noisy orders received by the bank and of the speculative orders (which are not noisy if a deterministic desaggregation is used); they are its "own" private information; the noises relative to these orders may or not be observed by the bank.*

The assumption neglecting news about the economic situation is made because the purpose of the paper is to investigate how the fact that each bank does not observe the noise relative to its clients' orders may be an explanation for the observed conditional heteroscedasticity (see appendix A). By contrast, the assumptions made about the private information have to be justified with respect to our goal.

About neglecting information due to quantities

1) The analysis of the effect of taking into account the information corresponding to the quantities, i. e. the demand for currency ask to the quoting bank by the remaining banks of the market, can be illustrated by studying how the information structure that a bank uses to determine its expectations evolves with the time from initial time to current time.

At time $t = 1$, each bank $j = 1$ to B of the market determines its expectation which is, among other variables, a function of its *own* private information i_1^j due to its clients orders. No bank has any information about the private information of the other ones. The quoting bank $\theta(1) = 1$ determines its quote which is a function of its expectation and therefore depends on its information structure. After this quote is known by the market, each of the other banks determines its own demand for currency and the total demand for foreign currency D_1^1 is asked to the quoting bank which is the only bank to know it. This demand is an aggregated indicator of the private information of the non-quoting banks $(j = 2 \text{ to } B)$.

At time $t = 2$, the private information of bank 1 contains now its own private information i_2^1 due to its clients' orders to be executed at that time and of the demand D_1^1.

Then its demand to the quoting bank $\theta(2) = 2$ is a function of its private information which consists of i_2^1 and D_1^1. Nevertheless, the private information D_1^1 will be conveyed through the demand for currency of bank 1 to the bank which is going to quote and after it has quoted, i. e. bank $\theta(2) = 2$ (the private information D_1^1 is not conveyed to the whole market but only to one bank). Then the quoting bank $\theta(1) = 1$ integrates its private information D_1^1 in its information structure with a one period lag. All other bank $j = 2$ to B of the market determines their expectations which, among other variables, are a function of its own private information i_2^j; each of these banks has no information about the private information of the other ones (the quote of the quoting bank $\theta(2) = 2$ does not include D_1^1, because bank 2 knows the demand asked by the other bank, including that of bank 1, only once it has been quoting).

This process carries on until all the bank have quoted once.

At time $t = 1 + B$, the bank $\theta(1) = 1$ is quoting for the second time. Since at time $t = 2$ its information set already included D_1^1, its information set and consequently its quote now includes D_1^1. The aggregate private information D_1^1 is conveyed by the quote to the market. Hence, at time $t > B$, the only own private information which has not been conveyed at all, is the own private information received at that time t by the $B-1$ banks which are not quoting and which will constitute the demand $D_t^{\theta(t)}$ asked to the quoting bank $\theta(t)$ once its quote is known. This is the loss of information for the market.

2) On the other hand, if each bank does not put the demand for currency in its information structure, then its expectations and consequently its quotes when it is quoting, or its demand for currency when it is not quoting, are independent of the quantities, e. g. when bank 2 does not put D_1^1 in its information structure to determine its demand at time 2. In this case, the bank makes its information structure coarser and its transactions do exist but are not informative. The private information is reduced to the own private information, i. e. the clients' orders. Only the quoting bank conveys its own private information to the market. For the other $B-1$ banks which have been quoting at the previous time $t-k$ with $k = 1$ to $B-1$, their private information respectively received at times t to $t-(k-1)$ are not conveyed to the market, e. g. at time t, the market has no clue of the private information received at times t, $t-1$, $t-(k-1)$ by the bank which has been

quoting at time $t-k$. This is the loss of information, for the market, due to not taking into account the quantities. It is more important than in the other way round.

Individual bank's knowledge about the noises relative to the commercial orders

The more knowledge each bank has about the noises relative to the orders, the finer its information structure. Three cases, ranked according to their decreasing informative character, can be considered:[12]

(i) Each bank observes all the noises relative to the clients' orders of all the banks; all banks are fully informed; it is the modeller's point of view.

(ii) Each bank observes all the noises relative to its own clients' orders but not the noises of the clients' orders of all the other banks; it is the bank's point of view.

(iii) Each bank observes its orders but not the corresponding noises.

To test the conjecture stated in the Introduction, the modeller's point of view is not appropriate; it is not considered in this paper. The modelling of the information structures will be presented for cases (ii) and (iii).

Whatever banks observe or not the noises relative to their own clients' orders, they do not observe the daily noises, which are the sum of bank's noises over the banks and the time. In that case, the only public information about the previous days consists of the past values of the fixing exchange rate. To simplify the analytical study of the dynamics of the OTC exchange rate, it is convenient to reduce the lack of information of any bank to the private information received by the banks during the opening of the current OTC market. The fixing exchange rate is assumed to be a linear function to make the information about daily noises to be inferred from the past values of the fixing exchange rate, i. e. the information generated by the daily noises is equal to the information generated by the fixing exchange rate. Each bank or speculator gets the information generated by the daily noises by observing the values of the fixing exchange rate.

Assumption 7 allows to discuss if the fact that each trader does not observe the noises relative to its clients' orders plays a key role for conditional heteroscedasticity to appear.

[12] A fourth case would be that of public information.

The extensive presentation of the bank's information structure which follows is motivated by the fact that when each bank does not observe the noises relative to its clients' commercial orders, private information is a non linear functions of these noises. Individual bank's information structures differ according to the fact it observes or not the noises relative to its clients' orders.

For all sources of information, uncertainty is modelled by the complete probability space $(\Omega, \mathbf{F}, \mathbf{P})$, where Ω is the set of the states of nature, ω, on which the tribe $\mathbf{F}$ and the probability measure $\mathbf{P}$ are defined. Then, to any source of information is associated the tribe generated by the set of variables of the source.

1. *Each bank does not observe the noises relative to its own clients' orders.*

(a) *Information structure due to the past values of the fixing exchange rate.*

According to assumption 7, the news about the economic situation represented by the set $\{\tilde{Z}_u; 1 \le u \le t-1\}$ of economic variables is not taken into account. At each fixing, the exchange rate is determined by the equilibrium condition which involves commercial and speculative orders: the former are exogenous random variables and the latter are determined by the information available to speculators: they consist of the past values of the fixing exchange rate $\{\tilde{e}_{D-K} \quad 1 \le K \le D-1\}$ which generates the tribe:

$$\mathbf{F1}_t^i = \mathbf{F1}_{D-1} = \sigma(\tilde{e}_{D-K}, 1 \le K \le D-1) \qquad i = 1 \text{ to } B \text{ and } t = 1 \text{ to } B\ P.$$

This information does not change during the day.

(b) *Information structure due to the orders of the bank's clients (private information).*

The set $\{\tilde{M}_u^{if}, \tilde{X}_u^{id}, S\tilde{P}E_u^{if}; 1 \le u \le t\text{-}1\}$ of orders received by bank i up to time t, where the condition $u \le t-1$ indicates that the orders to be executed at time t are known when the bank determines its second information structure at t.[13] Nevertheless, instead of considering the sequence of the orders of its clients, a bank can take into account the sequence of its implicit exchange rates, i.e. the exchange rates which clears the cumulative orders. The implicit exchange rate of bank i at time t reads:

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[13] In probabilistic terms, it means that the orders are predictable stochastic processes, i. e. $\mathbf{F2}_{t-1}^i$-measurable.

$$\tilde{e}_t^{*i} = \sum_{\tau=1}^{t} \tilde{X}_\tau^{id} \Bigg/ \left(\sum_{\tau=1}^{t} \tilde{M}_\tau^{if} + \sum_{\tau=1}^{t} S\tilde{P}E_\tau^{if} \right), \quad i = 1 \text{ to } B \text{ and } t = 1 \text{ to } B\ P$$

and it generates the information structure modelled by the tribe:

$$F2_t^i = \sigma\left(\tilde{e}_u^{*i} ; 1 \le u \le t-1\right) \qquad i = 1 \text{ to } B \text{ and } t = 1 \text{ to } B\ P.$$

Since the speculative orders received by bank i at time t are derived from a desaggregation of the speculators' demand for currency which is a function of the past values of the fixing exchange rate, the information structure $\mathbf{F2}_t^i$ is more informative than $\mathbf{F1}_{D-1}$:

$$\mathbf{F1}_{D-1} \subset \mathbf{F2}_t^i \quad i = 1 \text{ to } B \text{ and } t = 1 \text{ to } B\ P.$$

(c) *Information structure due to the previous quotes within the current intradaily period.* [13]

Each quote of the set $\left\{e_\tau = e_\tau^{\theta(\tau)}, \tau = 1 \text{ to } t-1\right\}$ of previous quotes depends on the corresponding information $\mathbf{F2}_\tau^{\theta(\tau)}$, i. e. it includes all the information available to the quoting bank $\theta(\tau)$. The information structure $\mathbf{F2}_t^i$ of bank i at time t is enlarged by the set $\left\{\tilde{e}_u = \tilde{e}_u^{\theta(t')}, 1 \le u \le t-1\right\}$ of previous quotes, which generates the tribe: $i = 1$ to B and $t = 1$ to $B\ P$

$$\mathbf{F3}_t^i = \mathbf{F3}_t = \sigma\left(\tilde{e}_u = \tilde{e}_u^{\theta(u)}, 1 \le u \le t-1\right) = \sigma\left(\bigcup_{\tau=1}^{t-1} \mathbf{F2}_\tau^{\theta(\tau)}\right)$$

Finally, the information structure of bank i at time t is given by the union of the three previous information structure, i. e. the tribe:

$$\mathbf{F}_t^i = \sigma\left(\mathbf{F}_{D-1} \cup \mathbf{F2}_t^i \cup \mathbf{F3}_t\right) \qquad i = 1 \text{ to } B \text{ and } t = 1 \text{ to } B\ P. \qquad (16)$$

Note that the private information that each bank has received is included in $\mathbf{I2}_t^i$ while the one which have been conveyed by the previous quotes is included in $\mathbf{I3}_t$. [14]

[13] We leave aside the technical question whether banks use information prior to the opening of the fixing or not. For the sake of simplicity, we provisionally assume that banks have no memory of what happened during the intradaily periods of the previous days.

[14] Since its last quote if it has already been quoting or since the opening of the OTC market if it has not already been quoting.

2. *Each bank observes the noises relative to its own clients' orders.*

(a) *Information structure due to the noises of the previous fixing dates*

Recall that, as stated in assumption 7, the fixing exchange rate is a linear function of the daily noises. This information is assumed to be inferred from the past values of the fixing exchange rate. This information does not change during D-day:

$$\mathbf{I1}_t^i = \mathbf{I1}_{D-1} = \sigma\left(\tilde{v}_{D-K},\ K = 1 \text{ to } D-1\right) \qquad i = 1 \text{ to } B \text{ and } t = 1 \text{ to } B\ P.$$

(b) *Information sub-structure due to the orders of the individual bank's clients (private information).*

The private information of bank i gathered up to time t consists of the set $\left\{\tilde{v}_\tau^{id},\ \tilde{v}_\tau^{if},\ \tau = 1 \text{ to } t\right\}$ of noises relatives to the commercial orders and which generates the tribe:

$$\mathbf{I2}_t^i = \sigma\left(\tilde{v}_\tau^{id},\ \tilde{v}_\tau^{if},\ \tau = 1 \text{ to } t\right) \qquad i = 1 \text{ to } B \text{ and } t = 1 \text{ to } B\ P.$$

(c) *Information structure due to the previous quotes*

Each quote of the set $\left\{e_\tau = e_\tau^{\theta(\tau)},\ \tau = 1 \text{ to } t-1\right\}$ of previous quotes includes all the information available to the quoting bank $\theta(\tau)$, i. e. $\mathbf{I2}_\tau^{\theta(\tau)}$. The public information due to the previous quotes generates the following tribe the explicit expression of which is given in proposition 8:

$$\mathbf{I3}_t^i = \mathbf{I3}_t = \sigma\left(\bigcup_{\tau=1}^{t-1} \mathbf{I2}_\tau^i\right) \qquad i = 1 \text{ to } B.$$

Finally, the information structure of bank i at time t is given by the union of the three previous information structure, i. e. the tribe:

$$\mathbf{I}_t^i = \sigma\left(\mathbf{I1}_{D-1} \cup \mathbf{I2}_t^i \cup \mathbf{I3}_t\right) \qquad i = 1 \text{ to } B \text{ and } t = 1 \text{ to } B\ P. \tag{18}$$

and it can be stated:

PROPOSITION 1: *Bank i's information structure reads:*

1. *If no bank observes the noises relative to its own clients' orders:*

$$\mathbf{F}_t^i = \sigma\left(\mathbf{F1}_{D-1} \cup \mathbf{F2}_t^i \cup \mathbf{F3}_t\right) \qquad i = 1 \text{ to } B \text{ and } t = 1 \text{ to } B\,P. \qquad (16)$$

2. *If each bank observes the noises relative to its own clients' orders.*

$$\mathbf{I}_t^i = \sigma\left(\mathbf{I1}_{D-1} \cup \mathbf{I2}_t^i \cup \mathbf{I3}_t\right) \qquad i = 1 \text{ to } B \text{ and } t = 1 \text{ to } B\,P. \qquad (18)$$

♦ See the previous discussion.♦

Note that the information due to the past values of the fixing exchange rate are included in the two other ones.

Two remarks can be made:

(i) The private information F2 being defined by a non linear function of the corresponding commercial orders might be less informative than the information contained in the noises. If a linearisation is made then both informations are equivalent and finally $\mathbf{F}_t^i = \mathbf{I}_t^i$ $\forall\ i = 1$ to B and $\forall\ t = 1$ to $B\,P$, because the three components of these tribes are identical.

(ii) An *observer* of the OTC market would not have any private information. Its information structure reads:

$$\Phi_t = \sigma\left(\mathbf{F1}_{D-1} \cup \mathbf{F3}_t\right) \qquad t = 1 \text{ to } B\,P.$$

B. Bank's optimal behaviour

As already mentioned, it is assumed, on the one hand:

ASSUMPTION 8: *The transactions between two banks are bounded in terms of foreign currency*: $\left|q^f\right| \le q^f *$,

where q^f is a transaction in foreign currency and $q^f *$ its bound. In the OTC market each bank trades with commercial traders and non-bank speculators and with some other banks.

And on the other hand:

ASSUMPTION 9: *Each bank (i) is risk neutral, (ii) acts myopically and not strategically.*

Consider a domestic bank. At time t, within the intraday period, the information structure of bank i is either $\mathbf{F}_t^i$ or $\mathbf{I}_t^i$, according assumption 7. For the general presentation given in this paragraph, denote the information structure by Φ_t^i.

At time t, within the intraday period, bank i maximises the expected value, conditional upon its information structure Φ_t^i, of its wealth $\tilde{W}_D^{id}$ at the forthcoming fixing D. $\tilde{W}_D^{id}$ is valued in domestic currency and bank i thus determines (i) its demand for foreign currency d_{t+k}^{if} $(k \geq 0)$, when it is not quoting and (ii) its quote e_{t+k} $(k \geq 0)$, when it is quoting. Because the quote of the quoting bank is but the OTC market exchange rate prevailing at the same time, the bank's label is omitted, to notify this value is a market one. Let T^i be the set of bank i quoting times prior to time t, i. e.:

$$T^i = \{t' : \theta(t') = i \ \ and\ t' < t\};$$

the bank's optimising problem reads:

$$\underset{d^{if}_{t+k \notin T^i},\, e_{t+k \in T^i}}{Max} \left\{ E\left[\tilde{W}_D^{id} \,\middle|\, \Phi_t^i \right] \right\} \tag{19a}$$

subject to:[15]

$$\begin{aligned} \tilde{W}_D^{id} = W_t^{id} + {}^f W_{t-1}^{if}\left(\tilde{e}_D - \tilde{e}_t\right) &+ \sum_{t+k \notin T^i}\left(\tilde{d}_{t+k}^{if} + T\tilde{O}I_{t+k}^{if}\right)\left(\tilde{e}_D - \tilde{e}_{t+k}\right) \\ &+ \sum_{t+k \in T^i}\left(\tilde{Q}_{t+k}^{if} + T\tilde{O}I_{t+k}^{if}\right)\left(\tilde{e}_D - \tilde{e}_{t+k}\right) \end{aligned} \tag{19b}$$

with:

$$T\tilde{O}I_t^{if} = \tilde{X}_t^{id} / \tilde{e}_t - \left(\tilde{M}_t^{if} + S\tilde{P}E_t^{if}\right) \tag{15}$$

$$W_t^{id} = {}^d W_{t-1}^{id} + {}^f W_{t-1}^{if}\, \tilde{e}_t$$

$${}^f W_t^{if} = {}^f W_{t-1}^{if} + \left(\tilde{d}_t^{if} + T\tilde{O}I_t^{if}\right) \qquad \text{if the bank is not quoting at } t > t_l^i,$$

$${}^f W_t^{if} = {}^f W_{t-1}^{if} + \left(\tilde{Q}_t^{if} + T\tilde{O}I_t^{if}\right) \qquad \text{if the bank is quoting at } t = t_l^i,$$

where W_t^{id} is total wealth of bank i at time t denominated in domestic currency d, ${}^d W_{t-1}^{id}$ wealth of bank i held, at time $t-1$, in domestic currency and denominated in

[15] The symbols in the sums of the RHS of equation (10b) mean: (i) in the first sum in the RHS $t+k \notin T^i, k \geq 0$ denotes the times at which the bank i is not quoting (i. e. is a "price taker"); (ii) in the second sum in the RHS $t+k \in T^i, k \geq 0$ denotes the times at which the bank i is quoting (i. e. is a "price maker").

domestic currency, ${}^{f}W_{t-1}^{if}$ wealth of bank i held, at time $t-1$, in foreign currency f and denominated in foreign currency, $\tilde{d}_{t+k}^{if}$ bank's demand for foreign currency, $\tilde{e}_D$ the exchange rate at the forthcoming fixing D, $\tilde{e}_{t+k}$ the exchange rate at the forthcoming intraday times at which the bank is quoting, $T\tilde{O}I_t^{if}$ is the total of orders of bank i to be executed at time t, defined as the number of units of foreign currency against the domestic one that bank i trades with its clients,[16] $\tilde{e}_t$ is the OTC exchange rate at time t, t_l^i is the last quoting time of bank i and $\tilde{Q}_t^{if}$ the transactions the bank i has to accept when it is quoting, i. e. the sum of the other banks' demand:

$$\tilde{Q}_t^{if} = -\sum_{j \neq i=1}^{B} \tilde{s}_t^{jf} \tag{20}$$

where $\tilde{s}_t^{jf}$ is the change in the bank j position in foreign currency at time t (see proposition 3).

Remember that in the budget constraint (19b) the clients' orders $T\tilde{O}I_{t+k}^{if}$ do not depend on the current exchange rate in the intraday period but on exchange rates at fixings. The optimal quote and demand for domestic currency are given by the following propositions:

PROPOSITION 2: *When a domestic bank is a* "price maker", *under the* assumption of an infinite elasticity of demand $\tilde{Q}_t^{if}$, *its optimal quote is*:[17]

$$\tilde{e}_t^i = \tilde{e}_t = E\left[\tilde{e}_D \middle| \Phi_t^i\right], \quad i = \theta(t) \tag{21a}$$

and the change in the bank's position in foreign currency is:

$$\tilde{s}_t^{if} = \tilde{Q}_t^{if} + T\tilde{O}I_t^{if}. \tag{21b}$$

♦ See appendix C.1.♦

The consequence of assuming an infinite elasticity of demand is that the individual bank's expected aggregated demand for currency is not taken into account.

PROPOSITION 3: *When a domestic bank is a* "price taker", *its optimal demand for the foreign currency, is:*

$$\tilde{d}_t^{if} = Sign\left\{E\left[\tilde{e}_D \middle| \Phi_t^i\right] - \tilde{e}_t\right\} q^{f*}, \quad i \neq \theta(t) \tag{22a}$$

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16 At the fixing time D, commercial traders and speculators can not have any order to be executed. All the orders, as already discussed (see section 2), are assumed to be independent of the current exchange rate $\tilde{e}_t$. Relaxing such an assumption, i.e. allowing for commercial traders and speculators to pass orders at the fixing would not modify our results but would complicate the presentation.

17 Recall that because at all time t, within the intraday period, the quote of the quoting bank is the OTC market exchange rate prevailing at that time, the bank's label is omitted, to notify this value is a market one.

and the change in the bank's position in foreign currency is:

$$\tilde{s}_t^{if} = \tilde{d}_t^{if} + T\tilde{O}I_t^{if} \qquad (22b)$$

♦ See appendix C.2. It results from the fact that the objective function in bank i program is linear in its arguments. ♦

Remark: The bank's decision is a best response.

Foreign banks maximise the conditional expectation of their wealth denominated in their "domestic" currency which is the foreign currency f. Their risky asset is the domestic currency d. The optimising problem of a foreign bank is thus similar to the previous one although conditional expectations are now relative to the inverse of the exchange rate $\left(1/e_t = E\left[1/\tilde{e}_D \middle| \Phi_t^i\right]\right)$. For analytical tractability, the optimal behaviour of the foreign bank is determined by a linearisation of its optimal program (see appendix C.3). The optimal behaviour of foreign banks is then identical to the one of domestic banks and Propositions 1 and 2 will be used, for domestic banks as well as for foreign ones.

4. THE OTC EXCHANGE RATE DYNAMICS: A THEORETICAL EXPLANATION FOR CONDITIONAL HETEROSCEDASTICITY

Two polar cases are successively considered: noises may be observable $\left(\Phi_t^{\theta(t)} = \mathbf{I}_t^{\theta(t)}\right)$ or not $\left(\Phi_t^{\theta(t)} = \mathbf{F}_t^{\theta(t)}\right)$. In any case, the determination of the OTC exchange rate dynamics is first presented and its conditional statistical properties are then given.

The conditional statistical properties of the OTC exchange rate depends on the information structure of the agent who is interested in. This agent can be either an external *observer* of the market or a bank. The difference between the information structures of these agents relies in the private information which is not in the observer's information structure which is reduced to the public information. This information structure has to be determined in the two cases under interest.

1. Each bank observes the noises relative to its own clients' orders

A. The determination of the OTC exchange rate dynamics

The OTC exchange rate, defined as $\tilde{e}_t = E\left[\tilde{e}_D \middle| \mathbf{I}_t^i\right]$, is determined by using the statistical properties of the conditioning variables and a bit of geometry in the relevant Hilbert spaces. The set of the random variables which represents the information of bank i at time t generates the tribe $\mathbf{I}_t^i$ (see section 3). Let Hv_t^i be the closed Hilbert sub-space of the random variables which are $\mathbf{I}_t^i$-measurable. Then $E\left[\tilde{e}_D \middle| \mathbf{I}_t^i\right]$ is the orthogonal projection of $\tilde{e}_D$ on Hv_t^i. Let Hv_{D-1} be the Hilbert sub-space of the random variables which are measurable with respect to the tribe generated by the past values of the daily noises (relative to the commercial orders), i. e. $\mathbf{Il}_{D-1} = \sigma\left(\tilde{v}_{D-K},\, K = 1 \; to \; D-1\right)$; it is a closed sub-space of Hv_t^i. The orthogonal projection of $\tilde{e}_D$ on Hv_{D-1} is $E\left[\tilde{e}_D \middle| \mathbf{Il}_{D-1}\right]$ which is also the orthogonal projection of $E\left[\tilde{e}_D \middle| \mathbf{I}_t^i\right]$ on Hv_{D-1} (by the law of iterated conditional expectations). Since $E\left[\tilde{e}_D \middle| \mathbf{Il}_{D-1}\right]$ is a time-invariant component of $E\left[\tilde{e}_D \middle| \mathbf{I}_t^i\right]$ belonging to the closed Hilbert sub-space Hv_t^i, then $E\left[\tilde{e}_D \middle| \mathbf{I}_t^i\right]$ is determined by the following three-step procedure:

(i) Specify a particular model to calculate $E\left[\tilde{e}_D \middle| \mathbf{Il}_{D-1}\right]$; the Broze, Gourieroux and Szafarz's (1989) model is here used (shortly BGS' model);

(ii) Calculate the implicit exchange rate that clears the commercial and speculative orders of bank i at time t, i. e. $\tilde{e}^{*i}_t$;

(iii) Use the orthogonality properties deduced from the rational expectations hypothesis and from the assumptions made for the stochastic components of the commercial and speculative orders.

(i) *The model of the fixing*

The fixing exchange rate and its conditional expectation is given by the following proposition.

PROPOSITION 4: *If assumptions* 1 *to* 9 *hold:*

1. *The value of the exchange rate at the forthcoming fixing reads:*

$$\tilde{e}_D = \mu_D + \sum_{K=0}^{D-1} \psi_K \, \tilde{v}_{D-K} ; \tag{23a}$$

2. *Its expectation conditional upon the information held in the past values of the daily noises writes:*

$$E\left[\tilde{e}_D \middle| \Pi_{D-1}\right] \equiv e^*_D = \mu_D + \sum_{K=1}^{D-1} \psi_K \, \tilde{v}_{D-K} ; \tag{24}$$

and the fixing exchange rate reads:

$$\tilde{e}_D = e^*_D + \psi_0 \, \tilde{v}_D \tag{23b}$$

3. *The first and second order unconditional moments of* $\tilde{e}_D$ *and its expectation* e^*_D *are:*

$$E\left[\tilde{e}_D\right] = E\left[e^*_D\right] = \mu_D \tag{25}$$

$$E\left[(e^*_D)^2\right] = \mu_D{}^2 + \sigma_v{}^2 \left(\sum_{K=1}^{\infty} \psi_K{}^2\right) \tag{26}$$

$$E\left[(\tilde{e}_D)^2\right] = E\left[(e^*_D)^2\right] + \psi_0{}^2 \, \sigma_v{}^2 \tag{27}$$

$$E\left[\tilde{e}_D \, e^*_D\right] = E\left[(e^*_D)^2\right] \tag{28}$$

where μ_D *is the deterministic component of the* $\tilde{e}_D$ *and* $\sigma_v{}^2$ *the variance of the daily noises.*

♦ See appendix D. ♦

The modulus of ψ_K is given by $(1+\beta\ \delta)^{-\frac{K}{2}}$; since β and δ are positive quantities, it converges to zero. *This result differs from the conclusion reached by BGS who obtained the non convergence*. Such a difference lies in the specification of the stochastic components of the process which is investigated. In the original BGS' model, the supply side of the market is approximated as a moving average of the noises; the current as well as the past values of the noises intervenes. In the present use of the BGS' model, the stochastic component which is introduced by the daily trade balance consists only of the current daily noise, see equation (7a). Hence the solution of the stochastic part of the fixing equilibrium condition determines the coefficient of zero order of the expansion of the fixing exchange rate:

$$\psi_0 = 1/\delta , \tag{29}$$

where δ is a function of the parameters of the model, equation (7d); while it is undetermined in the BGS' (1989) model. Then the OTC exchange rate, $\tilde{e}_{D-K}$, is a

$\sigma(\tilde{v}_{D-K-L}, K=1 \text{ to } D-1, L=0 \text{ to } D-K-1)$-measurable linear function. The information structure $\mathbf{F}\mathrm{I}_{D-1}$ reduces to the tribe generated by the random variables $(\tilde{v}_{D-K-L}, K=1 \text{ to } D-1, L=0 \text{ to } D-K-1)$, which reads:

$$\mathbf{F}v_{D-1} = \sigma(\tilde{v}_{D-K-L}, K=1 \text{ to } D-1, L=0 \text{ to } D-K-1)$$

and Hv_{D-1} denotes the closed Hilbert sub-space corresponding to these random variables.

(ii) *The implicit exchange rate*

The implicit exchange rate, i. e. $\tilde{e}*^i_t$, which clears the commercial and speculative orders of bank i at time t is obtained by writing down the corresponding equilibrium condition and subtracting it the fixing equilibrium condition, then $\tilde{e}*^i_t$ reads:

$$\tilde{e}*^i_t = e*_D + \psi_0 \frac{\bar{\tilde{v}}^i_t}{\bar{\lambda}^i_t} \tag{30a}$$

where $\bar{\tilde{v}}^i_t = \sum_{\tau=1}^{t}\left(\frac{\tilde{v}^{id}_\tau}{e_{M-1}} - \tilde{v}^{if}_\tau\right)$ is the cumulated noise and $\bar{\lambda}^i_t = \sum_{\tau=1}^{t}\lambda^i_\tau$ the cumulated shares of orders which have been order to bank i up to time t. Using equation (24), the implicit exchange rate of bank i at time t reads:

$$\tilde{e}*^i_t = \tilde{e}_D + \psi_0\left(\frac{\bar{\tilde{v}}^i_t}{\bar{\lambda}^i_t} - \tilde{v}_D\right). \tag{30b}$$

Consequently, if at time t bank i has a *representative sample* of orders of the market, the stochastic components of the commercial orders are in the same proportion than the corresponding deterministic part, then $\tilde{v}^{if}_t = \lambda^{if}_t\ \tilde{v}^m_D$, $\tilde{v}^{id}_t = \lambda^{if}_t\ \tilde{v}^x_D$ and after some algebra $\bar{\tilde{v}}^i_t = \bar{\lambda}^i_t\ \tilde{v}_D$ and the implicit and the fixing exchange rates are equal, i. e. $\tilde{e}*^i_t = \tilde{e}_D$.

PROPOSITION 5: *The difference between the implicit exchange rate and the expectation is orthogonal to the expectation* $e*_D$:

$$E\left[\left(\tilde{e}*^i_t - e*_D\right)e*_D\right] = \frac{\psi_0}{\bar{\lambda}^i_t} E\left[\bar{\tilde{v}}^i_t\ e*_D\right] = 0 \tag{31}$$

when $\overline{\lambda}_\gamma^i$ is deterministic as it is assumed in the present paper.[19]

♦ Proof is straightforward. ♦

The definition (30) of $\tilde{e}*_t^i$ and the orthogonality condition (31) show that the Hilbert sub-space H_t^i can be split into the (direct) sum of two sub-spaces corresponding respectively to $e*_D$ and the cumulative noise $\overline{\overline{v}}_t^i$:

$$H_t^i = H\eta_{D-1} \oplus Hv_t^i.$$

Then the previous property being used to construct the series of Hilbert sub-spaces from time 1 up to time t leads to the more general expression given in the following proposition:

PROPOSITION 6: *The Hilbert space H_t^i associated to the information hold by bank i at time t can be spanned into the direct sum of the closed Hilbert sub-spaces corresponding to $e*_D$ and the last B cumulative noises corresponding to the noises of the orders of the quoting bank and the $B-1$ noises included in the $B-1$ last OTC exchange rate values* $\left(\overline{\overline{v}}_t^{\theta(t)}, \overline{\overline{v}}_s^{\theta(s)}; s = t-(B-1) \text{ to } t-1\right)$:

$$H_t^i = H\eta_{D-1} \oplus Hv_{t-(B-1)}^{\theta(t-(B-1))} \oplus \ldots \oplus Hv_t^{\theta(t)}. \quad (32)$$

♦ See appendix E. ♦

Since H_t^i can be written as in equation (32), the OTC exchange rate may be written as a sum of the corresponding variables as it is going to be proved.

(iii) *Taken into account the past values of the quotes*

Because each bank includes its private information in its quote, then it is intuitive that at time t when a bank i takes into account the quote of bank j at time $s < t$, it increases the Hilbert sub-space H_t^i relative to its information space by one dimension corresponding to the private information of bank j, e. g. Hv_s^j and which is orthogonal to its information space H_t^i, according to the meaning of equations (32).

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[19] If the λ's are stochastic, $\overline{\lambda}_\gamma^i$ has to be replaced by $E\left[\overline{\lambda}_\gamma^i\right]$.

The sequence of the H's is obtained by considering first the construction of $H_1^{\theta(1)}$, then $H_2^{\theta(2)}$, and so on up to time t. At time t, the expectation of the forthcoming fixing exchange rate conditional upon its information structure of the quoting bank $\theta(t)$ reads as given in the following proposition:

PROPOSITION 7:

1. *Under orthogonality assumption of noises (assumption 3) and properties of the implicit exchange rate (proposition 5), the OTC exchange rate dynamics is given by the following expression:*

$$\tilde{e}_t \equiv E\left[\tilde{e}_D \middle| \mathbf{I}_t^{\theta(t)}\right] = e^*{}_D + \psi_0 \sum_{k=0}^{Max(t-1,\, B-1)} \tilde{\tilde{v}}_{t-k}^{\theta(t-k)} \qquad (33)$$

where ψ_0 is given by equation (29).

2. *The change of the OTC exchange rate is proportional to the change in the private information, i. e. the cumulated commercial orders, of the quoting bank since its previous quote; it reads:*

$$\begin{aligned} \tilde{e}_t - \tilde{e}_{t-1} &= \psi_0\, \tilde{u}_t \\ \tilde{u}_t &= \tilde{\tilde{v}}_t^{\theta(t)} - H(t-B)\, \tilde{\tilde{v}}_{t-B}^{\theta(t)} \end{aligned} \qquad t = 1, 2, \ldots, B\,P. \qquad (34)$$

where $H(x)$ is the Heaviside function $H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$

3. *The OTC exchange rate is a random walk as soon as t exceeds B.*

4. *The value of the OTC exchange rate at the last quoting time, i. e. $t = B\ P$, is as close to the fixing exchange rate as the missing private information is insignificant, e. g. the number of quoting time is much greater than one $(B\ P >> 1)$:*

$$\tilde{e}_{BP} \to \tilde{e}_D \quad \text{if } B\ P >> 1.$$

♦ See appendix E. ♦

B. The conditional statistical properties of the OTC exchange rate

The conditional statistical properties of the OTC exchange rate can is analysed from the observer's points of view. Because the non quoting banks are in the same situation than the observer, bank's analysis is presented in, appendix E, § A.6 and § B.[20]

At any time, the observer's information structure is the tribe Φ_{t-1} generated by the public information which consists of the past values of the fixing and of the OTC exchange rates; hence:

$$\Phi_{t-1} = \sigma\left(\sigma\left(\tilde{e}_{D-K};\ 1 \le K \le D-1\right) \cup \sigma\left(\tilde{e}_{t-k};\ 1 \le k \le t-1\right)\right).$$

The observer's information structure and the conditional statistical properties he/she obtains for the OTC exchange rate are given in the following proposition:

PROPOSITION 8:

1. *The observer's information structure is due to the vector of noises which have been conveyed by the quoting banks to the market; it reads:*

$$\Phi_{t-1} = \sigma\left(\sigma\left(\tilde{V}_{t-k}^{\theta(t-k)};\ 3 \le k \le t-1\right) \cup \mathrm{I\!I}_{D-1}\right) \equiv \mathrm{I}3_{t-1} \qquad t = 2, \ldots, B\ P$$

where $\tilde{V}_{t-k}^{\theta(t-k)} = \left(\tilde{v}_{t-k}^{\theta(t-k)}, \ldots, \tilde{v}_{\max(t-k,\,t-k-(B-1))}^{\theta(t-k)}\right)$

and $\mathrm{I\!I}_{D-1} = \sigma\left(\tilde{v}_{D-K},\ 1 \le K \le D-1\right).$

2. *The OTC exchange rate is a random walk, the conditional expectations and variance of which respectively reads for* $t = 2, \ldots, B\ P$:

(i) *Its conditional expectations is:*

$$E\left(\tilde{e}_t \middle| \Phi_{t-1}\right) = e_{t-1} \tag{35}$$

(ii) *Its conditional variance is:*

$$Var\left(\tilde{e}_t \middle| \Phi_{t-1}\right) = \psi_0^{\ 2}\ \sigma_v^{\ 2}\ \min(t,\ B) \tag{36}$$

---

[20] Consider a non-quoting bank. Because of the assumption of statistical independence between the orders of different banks, its private information does not give to the bank any kind of information of the private information of the bank which is going to be quoting at time t. Then this private information is independent of any non-quoting bank's information structure. All the non-quoting banks are in the observer's situation with respect to the noises corresponding to the private information the quoting bank. By contrast, the quoting bank $\theta(t)$ knows its private information which is going to be conveyed to the market by its quote, at time t. Then the banks' conditional properties are given in the following proposition:

Then the OTC exchange rate is a martingale, as soon as each bank has quoted once, $t > B$.

♦ Part 1, see appendix E. Part 2, take the definition (34) of the OTC exchange rate. The noises $\tilde{u}_t$ are not observed by the observer; they are independent of Φ_{t-1}; then take the conditional expectations and variance moment of the equation (34) to obtain the result.♦

If the orthogonality assumption made for the order noises while this assumption is relaxed while the BGS framework is kept (to model the equilibrium of the market at the fixing and to determine the banks' implicit exchange rates), then the OTC exchange rate can no more be written as in equation (33). Nevertheless, the method used to obtain expression (33) can be used once the Gram-Schmidt orthogonality procedure is used. The OTC exchange rate would be defined by an expression similar to equation (33), but the coefficient would be no more unitary; it would be determined by second order moments of noises which would be deterministic if these moments are known (it would be the case if the joint distribution of the noises is common knowledge, e. g. multivariate normal distribution.

2. No bank observes the noises relative to its own clients' orders: an OLSQ estimation.

The determination of the OTC exchange rate dynamics by OLSQ is first given, its conditional statistical properties are then presented.

A. The determination of the OTC exchange rate dynamics

Bank $\theta(t)$ when quoting at time t knows:

(i) The sequence of past values of the exchange rate at the fixing $\left(e_{D-K};\, 1 \leq K \leq D-1\right)$.

(ii) The sequence of its implicit exchange rates $\left(e^{*\theta(t)}_{t-k}\right)_{k=0 \text{ to } t-1}$ which characterises its private information.

(iii) The sequence of the previous quotes $\left(e_{t-k}\right)_{k=1 \text{ to } t-1}$.

The first thing that bank $\theta(t)$ has to do is to get a good estimator $\hat{e}^*_D$ of the expectation e^*_D of the exchange rate at the forthcoming fixing. The discussion about the choice of the estimator is beyond the scope of this paper. However, note that as far as the OTC exchange rate is concerned, $\hat{e}*_D$ is analogous to a constant over the opening time of the OTC market; it is a constant piecewise function. This does not mean it is a pure constant, since the determination of the $\hat{e}*_D$ depend on the speculators' demand for currency which on the one hand is a function of the past. $\hat{e}*_D$ can be replaced by the expectations $e*_D$ given by the BGS model.

Note that bank $\theta(t)$ can limit itself to considering its last implicit exchange rate $e*_t^{\theta(t)}$ which sums up all its private information. However, part of its private information has already been conveyed to the market by its previous quotes (between time $t = 1$ to time $t = t - B$). To take this fact into account the implicit exchange rate $e*_{t-B}^{\theta(t)}$ should be included in the information set of bank $\theta(t)$. Similarly:

PROPOSITION 10: *According to the previous analysis of the available information to the quoting bank, the OTC exchange rate is set equal to the OLSQ estimator that each bank determines for its quoting time from the observations of the previous days. Then at time t of D-day:*

$$e_t = \hat{a}_t \; \hat{e}*_D + \hat{b}_{0,t} \; e_{t-1} + \hat{b}_{1,t} \; e*_t^{\theta(t)} + \hat{c}_t \qquad \text{if } t \le B \qquad (37a)$$

$$e_t = \hat{a}_t \; \hat{e}*_D + \hat{b}_{0,t} \; e_{t-1} + \hat{b}_{1,t} \; e*_t^{\theta(t)} + \hat{b}_{2,t} \; e*_{t-B}^{\theta(t)} + \hat{c}_t \qquad \text{if } t > B. \qquad (37b)$$

where the right hand side of these equations are the OTC estimators. The set of parameters is the solution of a linear system.

If $e*_t^{\theta(t)}$ and $e*_{t-B}^{\theta(t)}$ are determined by equation (30a), then the variances and covariance's involved in the linear system are analytically determined and the theoretical values of parameters are obtained, e. g. for $t > B$:

$$\hat{a}_t = -\left(\bar{\lambda}_t^{\theta(t)} - \bar{\lambda}_{t-B}^{\theta(t)}\right), \quad \hat{b}_{0,t} = 1, \quad \hat{b}_{1,t} = \bar{\lambda}_t^{\theta(t)}, \quad \hat{b}_{2,t} = -\bar{\lambda}_{t-B}^{\theta(t)}, \quad \hat{c}_t = 0$$

verifying the following constraints:

$$\hat{a}_t = -\left(\hat{b}_{1,t} + \hat{b}_{2,t}\right), \quad \hat{b}_{0,t} = 1, \quad \hat{c}_t = 0.$$

♦ The results are obtained by straightforward calculations. ♦

Proposition 10 indicates that the quoting bank knowing the theoretical form of the equation determining the OTC exchange rate and that, at least asymptotically, the values of the estimated parameters converge to the theoretical ones.

B. The conditional statistical properties of the OTC exchange rate

At any time, the observer's information structure is the tribe generated by the public information, equation (35). As in case **A**, because of the assumption that the fixing exchange rate is a linear function of the daily noise, he/she is still able to infer the information given by the daily noises. By contrast, observing the past quotes does not give any information on the noises, since they are not observed by the banks. The issue is then the determination of $\mathbf{F3}_{t-1} \equiv \sigma\left(\tilde{e}_{t-k}; 1 \le k \le t-1\right) = \sigma\left(\bigcup_{k=1}^{t-1} \sigma\left(\tilde{e}_{t-k}\right)\right)$. Recall that when noises are not observed, the implicit exchange rate is a *non-linear* function of them and $\mathbf{F3}_{t-1}$ is different from $\mathbf{I3}_{t-1}$.

The observer's information structure and the conditional statistical properties he/she obtains for the OTC exchange rate are given in the following proposition:

PROPOSITION 11:

1. *The observer's information structure with respect to the definition of the OTC exchange rate, equations (37), reads:*

$$\Phi_{t-1} = \mathbf{F3}_{t-1} \equiv \sigma\left(\tilde{e}_{t-k}; 1 \le k \le t-1\right) = \sigma\left(\sigma\left(\tilde{U}_{t-k}^{\theta(t-k)}; 1 \le k \le t-1\right) \cup \mathbf{I1}_{D-1}\right).$$

where for $k = 1$ to $t-1$:

$$\tilde{U}_{t-k}^{\theta(t-k)} = \left(e^{*\theta(t-k)}_{t-k}, \ldots, e^{*\theta(tk1)}_{\max(t-k, t-k-(B-1))}, \hat{a}_{t-k}, \hat{b}_{i,t}; i = 0,1,2, \hat{c}\right).$$

2. *For the observer, according to the OLSQ estimation of the OTC exchange rate if* $t > B$:

(i) *Its conditional expectations is:*

$$E\left(\tilde{e}_t \middle| \Phi_{t-1}\right) = E\left(\hat{e}_t \middle| \Phi_{t-1}\right) \tag{38}$$

(ii) *Its conditional variance is:*

$$Var\left(\tilde{e}_t \middle| \Phi_{t-1}\right) = e*_D{}^2\ Var\left(\hat{a} \middle| \Phi_{t-1}\right) + e_{t-1}{}^2\ Var\left(\hat{b}_0 \middle| \Phi_{t-1}\right)$$
$$+ Var\left(\hat{b}\ e*_t^{\theta(t)}{}_1 \middle| \Phi_{t-1}\right) + e*_{t-B}^{\theta(t)2}\ Var\left(\hat{b}_2 \middle| \Phi_{t-1}\right)$$
$$+ 2\ e*_D\ e_{t-1}\ Cov\left(\hat{a}, \hat{b}_0 \middle| \Phi_{t-1}\right) + 2\ e*_D\ Cov\left(\hat{a}, \hat{b}_1\ e*_t^{\theta(t)} \middle| \Phi_{t-1}\right)$$
$$+ 2\ e*_D\ e*_{t-B}^{\theta(t)}\ Cov\left(\hat{a}, \hat{b}_2 \middle| \Phi_{t-1}\right)$$
$$+ 2\ e_{t-1}\ Cov\left(\hat{b}_0, \hat{b}_1\ e*_t^{\theta(t)} \middle| \Phi_{t-1}\right) + 2\ e_{t-1}\ e*_{t-B}^{\theta(t)}\ Cov\left(\hat{b}_0, \hat{b}_2 \middle| \Phi_{t-1}\right)$$
$$+ 2\ e*_{t-B}^{\theta(t)}\ Cov\left(\hat{b}_1\ e*_t^{\theta(t)}, \hat{b}_2 \middle| \Phi_{t-1}\right) \qquad (39)$$

♦ Part 1: see appendix D. Part 2: The results are obtained by straightforward calculations from equations (37). ♦

The expression (39) of the conditional variance indicates that:

(i) In the general case, the parameters, estimated by any bank using a standard econometric method, would have some variance, then e. g. the parameter $\hat{b}_0$ can be different from one, and the observer would observes that the *OTC exchange exhibits conditional heteroscedasticity*.

(ii) If any bank is able to get the theoretical values for the parameters, then the conditional variance is no more dependent on past values of the OTC exchange rate; *there is no conditional heteroscedasticity*.

The origin of the *conditional heteroscedasticity* is not in the functioning of the market but in the gap between, on the one hand, the necessary information to get the theoretical values of the second order moments which determine the coefficients (assuming the available econometric techniques have a sufficient accuracy), and on the other hand, the information embedded in the observations. If the observed variables are fully informative, then the conditional heteroscedasticity is expected to disappear.

5. CONCLUSION

This paper is a first attempt in suggesting a theoretical explanation to the conditional heteroscedasticity observed in time series of financial asset prices. The conjecture which is tested is that conditional heteroscedasticity depends on the accuracy with which each trader in the market -all are rational- observes his/her private information. In the forex market quantities are not common knowledge and it is realistic to consider that they are little informative; hence our conjecture has been tested under an unfavourable hypothesis. Two polar cases have been investigated: (i) each bank observes the noises relative to its clients' orders; (ii) no bank observes the noises relative to its clients' orders. The main results for an observer of the market (or a non-quoting bank) are:

1. The OTC exchange rate is a random walk, with a constant conditional variance, when each bank observes the noises relative to its clients' orders.

2. The OTC exchange rate exhibits some conditional heteroscedasticity when no bank observes the noises relative to its clients' orders.

Throughout the paper some particular points which might be interesting in some other field of research have been considered:

1. In the model two time scales corresponding to two type of behaviour have been introduced. The largest one is characteristic of the speculator's behaviour who is risk averse. A rationale is that investigating for a much longer period than a day, he is not aware of intraday fluctuations. The smallest one is the order of magnitude of the time interval between two successive quotes in the OTC market. Over this time scale, banks are risk neutral.

2. The interaction between banks has been explicitly introduced in the determination of the bank's expectations to take into account of the information conveyed by the quotes. A geometric technique, in Hilbert spaces, is proposed to obtain the optimal expression of the OTC exchange rate; it is quite general. It can be employed every times a model contains two time scales such that the dynamics of an observed variable is modelled on the large scale and that an agent, or the modeller, is interested in the evolution of an observed variable on the smaller scale.

3. If noises are not observed, conditional heteroscedasticity appears when the coefficients of the econometric model used be any bank to determine an estimator of the OTC exchange rate, are not deterministic. Since these coefficients are functions of second order moments, conditional heteroscedasticity is somewhat related to non ergodicity of second order moments.

Further developments can be done by relaxing some assumption. For example, (i) By introducing the quantities to determine the expected demand for currency that the quoting bank faces and by taking into account the informative character of the quantities. (ii) By assuming the banks have heterogeneous horizons. (iii) By suppressing the clearing condition of the bank's position in foreign currency. (iv) The non revision of expectations by speculators can be dropped. Speculators could be assumed to revise their expectations by a procedure analogous to the individual bank's one.

Market segmentation is a question often debated in the literature; simulations will also be undertaken to investigate the occurring of sub-markets with respect to the correspondents' network structure. Such an issue is discussed when generating a bank's network either by random drawing from the banks' set or by using random graphs (Kirman, Oddou and Weber, 1986 and Ioannides, 1990). All the developments are expected to be to difficult to be solved analytically; simulations should be needed.

References

Admati, A. R. (1987). "Information in Financial Markets: The Rational Expectations Approach" in *"Financial Markets and Incomplete Information"*, S. Bhattachrya and G. M. Constantinides, Editors. Savage, Rowman and Littlefield. pp. 139-52.

__________ and Pfleiderer P. (1988). "A Theory of Intraday Patterns: Volume and Price Variability", *Review of Financial Studies*, vol. 1, no 1, pp. 3-40.

Azariadis, C. (1981). "Self-fulfilling Proficiencies", *Journal of Economic Theory*, vol. 25.

_________ and Guesnerie, R. (1982). "Prophecies créatrices" et persistance des théories", *Revue Economique*, vol. 5.

Bollerslev, T. (1986). "Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics*, vol. 31, pp. 307-27.

_________, Chou R., and Kroner, K. (1992). "ARCH Models in Financee: a Review of the *Theory and Empirical Evidence", Journal of Econometrics, vol. 52*, pp. 5-60.

Bray, M. (1985). "Rational Expectations, Information and Asset Markets: An Introduction", Oxford Economic Papers, vol. 37, no. 2, pp. 161-95.

Broze, L., Gourieroux, C. and Szafarz, A. (1989). "Speculative bubbles and exchange of information on the market of a storable good", in "Economic complexity: *Chaos,* *Bubbles and Nonlinearity*", W. A. Barnett, J. Geweke and K. Shell, Editors. Cambridge: C. U. P.

Chauveau, T. (1990). "Optimal Monetary Policies in a Small Open Economy", in *Monetary Policy*, Artus, P. and Barroux, Y., eds, Amsterdam: Kluwer Academic Press.

Dacorogna, M. M., Müller, U. A., Nagler, R. J., Olsen, R. B. and Pictet, O. V. (1993). "A geographical model for the daily and weekly seasonal volatility in the foreign exchange market", *Journal of International Money and Finance*, vol. 12, 413-38.

Duffie, D. (1988). *Security Markets - Stochastic Models*, New York: Academic Press.

________ (1992). *Dynamic Asset Pricing Theory*, Princeton: Princeton University Press.

Grossman, S. J. (1976). "On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information", *Journal of Finance*, vol. 31, pp. 573-85.

____________ (1981). "An Introduction to the Theory of Rational Expectations Under Asymmetric Information", *Review of Economic Studies*, vol. 48, pp. 541-60.

Ioannides, Y. M. (1990). "Trading Uncertainty and Market Form", *International Economic Review*, vol. 31, no. 3, pp. 619-38.

Kirman, A. (1993), "Ants, Rationality, and Recruitment", *Quarterly Journal of Economics*, vol. CVIII, no. 1, pp. 137-56.

_________, Oddou, C. and Weber, S. (1986). "Stochastic Communication and Coalition Formation", *Econometrica*, vol. 54, pp. 129-38.

Magee, S. (1973), "Currency Contracts, Pass Trough and Devaluation", *Brookings Papers on Economic Activity*, no 1.

Mandelbrot, B. (1963). The variation of Certain Speculative Prices", *Journal of Business*, vol. 36, pp. 394-419.

Muth, J. F., (1961), "Rational Expectations and the Theory of Prices Movements", *Econometrica*, vol. 29, pp. 315-35.

Nelson, D. B. (1990) "ARCH Models as Diffusion Approximations", *Journal of Econometrics,* vol. 4, pp 7-38.

Orléan, A. (1992). "Contagion des opinions et fonctionnement des marchés financiers", *Revue Economique*, forthcoming.

________ and Robin, J.-M. (1991). "Variability of Opinions and Speculative Dynamics on the Market of a Storable Good", mimeo, pp. 40.

Topol, R. (1991). "Bubbles and Volatility of Stock Prices: Effect of Mimetic Contagion", *Economic Journal*, vcl. 101, no. 407, pp. 786-800.

West, K. D. (1988) "Bubbles, Fads and stock price volatility tests: a Partial evaluation". *Journal of Finance*, vol. 43, no 3, pp. 639-55.

APPENDIX A

About conditional heteroscedasticity

To discuss conditional heteroscedasticity of the price of a financial asset is equivalent to analyse the effect of the observed values of its explanatory variables (or exogenous variables) on its conditional variance. The intuition underlying the concept of conditional heteroscedasticity can be recalled as following. The conditional variance of the price will be constant if it is a constant function of the set of values taken by each of the explanatory variables in any sample of the different variables over a given period of time. This mean that the explanatory variables produce a mean-value effect over the given period; this is the effect that would be produced by an infinite number of sample (mean values are realised, e. g. law of great number or ergodicity). By contrast, heteroscedasticity is exhibited every time at least one of the explanatory variable which makes the conditional variance of the price as a non constant function of its values, i. e. it takes a set of values which is not sufficient to produce the mean-value effect to make the conditional variance independent of the sample.

The price of a financial asset results from agent's decision conditional to his/her information which have a public and a private components.

As far as the public information is concerned, the "macroeconomics news", e. g. unemployment, budget deficit, are such that the time interval between two successive of its published values (month) is much greater than the price sampling period (day). Then this kind of variables do not take a sufficient number of values over the period to realise the mean-value effect; intuitively, they are a cause for conditional heteroscedasticity. Any variables of this kind will produce conditional heteroscedasticity.

As far as the private information is concerned, the speed with which the agent conveys his/her private information to the market will intuitively be a cause for conditional heteroscedasticity. But this process consists of two steps which concern respectively the observation by the agent of its private information and the speed of its conveying to the market. If the intuition relative to the conditional heteroscedasticity suggests that a slow conveying emphasises the possibility for conditional heteroscedasticity,[1] it is not clear

[1] Recall that conditional heteroscedasticity, as an ARCH effect, is observed on short time scales (day) and not on longer ones (month).

about the role of the accuracy with which the private information is observed (first step). Such an analysis is the purpose of this paper.

APPENDIX B[1]

Optimal behaviour of the speculators

The behaviour of both domestic end foreign of speculators is characterised by the following assumption:

Assumption B.1.

The speculator is risk-averse. His behaviour is characterised by a CARA utility function U^k:

$$U^k\left(W^k\right) = -Exp\left(-b^k W^k\right),$$

where b^k is the speculator's risk aversion and W^k the speculator's wealth which is denominated in domestic (foreign) currency in the case of a domestic (foreign) speculator.

To take investment decisions, the speculator maximises the expected utility of his wealth at the forthcoming fixing D conditional upon his information. Recall that the speculator is assumed to take his investment decisions once for all day denoted[2] $\{(D-1,\ D)\}$, after the fixing at $D-1$ and before the opening of the OTC market. Let $\mathbf{F}^k_{D\text{-}1}$ be the k-speculator's information structure before the opening of the OTC market. Hence, the speculator *does not revise* his information structure and consequently *his expectations* during the period $\{(D-1,\ D)\}$, i. e. news arriving during the period are taken into account.

Assumption B.2.
Transactions costs are negligible.

The behaviour of a domestic speculator is modelled and that of a foreign speculator is then deduced.

Behaviour of the domestic speculator

The domestic speculator k maximises, before the opening of the OTC market, the expected utility of his terminal wealth $\tilde{W}^{kd}_D$ (in domestic currency) at the forthcoming fixing conditional upon its information structure $\mathbf{F}^k_{D-1}$. The domestic speculator's optimising problem where the foreign currency is the risky asset reads:[3]

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[1] When necessary look to the main text for the definitions of symbols.

[2] The short hand notation $\{(D-1,\ D)\}$, instead of $(D-1,\ D)\cap\mathbf{N}$, is chosen for the sake of simplicity.

[3] The speculator optimising problem is written as it is usually done in the portofolio theory Broze, Gourieroux and Szafarz ("Speculative bubbles and exchange of information on the market of a storable good", in "*Economic complexity: Chaos, Bubbles and Nonlinearity*", W. A. Barnett, J. Geweke and K. Shell, Editors. Cambridge: C. U. P., 1989), Merton (*Continuous-Time Finance*, Oxford: Basil Blackwell,

$$\underset{\{{}^f W_{D-1}^{kf}\}}{Max}\ E\left[U^k\left(\tilde{W}_D^{kd}\right)\middle|\mathbf{F}_{D-1}^k\right] \tag{B.1}$$

subject to the domestic speculator's budget constraint:

$$\tilde{W}_D^{kd} = W_{D-1}^{kd}\ R_{D-1}^d + {}^f W_{D-1}^{kf}\left(R_{D-1}^f\ \tilde{e}_D - R_{D-1}^d\ e_{D-1}\right) \tag{B.2}$$

with: $W_{D-1}^{kd} = {}^d W_{D-2}^{kd} + {}^f W_{D-2}^{kf}\ e_{D-1}$

and where:

e_D = exchange rate, $R_D^d = 1 + r_D^d$, $R_D^f = 1 + r_D^f$,

r_D^d = domestic interest rate (on D day and on a daily basis),

r_D^f = foreign interest rate (on D day and on a daily basis),

${}^d W_{D-1}^{kd}$ = wealth of speculator k held in domestic currency and denominated in domestic currency at the end of D day,[4]

${}^f W_{D-1}^{kf}$ = wealth of speculator k held in foreign currency and denominated in foreign currency at the end of D day,[5]

$\tilde{W}_D^{kd}$ = total wealth of speculator k denominated in domestic currency at the beginning of $D + 1$ day.[6]

If we want to write down the budget constraint in terms of the net foreign asset bought by speculator k at the beginning of D day,[7] ${}^f S_D^{kf}$, the dynamic equation of each kind of wealth is given by the following accounting equations:

$${}^d W_D^{kd} = {}^d W_{D-1}^{kd} - {}^f S_D^{kf}\ e_{D-1},$$
$${}^f W_D^{kf} = {}^f W_{D-1}^{kf} + {}^f S_D^{kf}.$$

Define the forecasting error $\tilde{\varepsilon}_D^k$ by:

$$\tilde{e}_D = E\left[\tilde{e}_D\middle|\mathbf{F}_{D-1}^k\right] + \tilde{\varepsilon}_D^k; \tag{B.3}$$

the conditional expectation of $\tilde{\varepsilon}_D^k$ is equal to zero and its variance is that of the exchange rate.

1990): the speculator clears his position in the foreign currency and buys a new amount of foreign currency after he knows the exchange rate; then his wealth in the foreign currency is determined at each period. An other presentation would be to assume the speculator does not determine his wealth in the risky asset but his flow of risky asset ${}^f S_{D-1}^{kf}$ for the period; his control is ${}^f S_{D-1}^{kf}$ and his budget constraint reads:

$$\tilde{W}_D^{kd} = {}^d W_{D-2}^{kd}\ R_{D-1}^d + {}^f W_{D-2}^{kf}\ R_{D-1}^f\ \tilde{e}_D + \left(R_{D-1}^f\ \tilde{e}_D - R_{D-1}^d\ e_{D-1}\right){}^f S_{D-1}^{kf}.$$

4 End of D day means ${}^d W_{D-1}^{kd}$ and ${}^f W_{D-1}^{kf}$ are components of the portofolio held by the speculator *after* he known e_{D-1} but *before* he knows e_D.

5 See previous footnote.

6 After e_D is known by the speculator.

7 After e_{D-1} is known by the speculator.

Define the expected gain from holding one unit of foreign currency one day g_D^{kf} by:

$$g_D^{kf} = E\left[\tilde{e}_D \middle| \mathbf{F}_{D-1}^k\right] - \rho_{D-1}\, e_{D-1} \tag{B.4}$$

with:

$$\rho_D = R_D^d / R_D^f = \left(1 + r_D^d\right) / \left(1 + r_D^f\right) \approx 1 + \left(r_D^d - r_D^f\right), \tag{B.5}$$

then the speculator's budget constraint (B.2) becomes:

$$\tilde{W}_D^{kd} = W_{D-1}^{kd}\, R_{D-1}^d + g_D^{kf}\, R_{D-1}^f\ {}^f W_{D-1}^{kf} + R_{D-1}^f\ {}^f W_{D-1}^{kf}\, \tilde{\varepsilon}_D^k . \tag{B.6}$$

Assumption B.4.

The distribution of $\tilde{e}_D$ conditional upon the information available to speculator k on day $D-1$ is normal.

The first order condition of the optimising problem of the domestic speculator gives his/her demand for foreign currency:

$${}^f W_{D-1}^{kf} = \frac{g_D^{kf}}{b^k\, (\sigma_D^k)^2\, R_{D-1}^f}, \tag{B.7}$$

where $(\sigma_D^k)^2$ is the conditional variance of $\tilde{\varepsilon}_D^k$, i. e. the variance of $\tilde{e}_D$ conditional upon the information available to speculator k on day $D-1$:

$$(\sigma_D^k)^2 = Var\left(\tilde{e}_D | \mathbf{F}_{D-1}^k\right). \tag{B.8}$$

Assumption B.5.

Common knowledge among speculators is assumed: their information consists of the tribe generated by the set of past values of the exchange rate at the fixing:

$$\mathbf{F}_{D-1}^k = \mathbf{F1}_{D-1} = \sigma\left(\tilde{e}_{D'},\ 0 \le D' \le D-1\right)\ \ \forall\ k .$$

Then the relevant variables defined by equations (B.4), (B.8) respectively become:

$$g_D^f = E\left[\tilde{e}_D \middle| \mathbf{F1}_{D-1}\right] - \rho_{D-1}\, e_{D-1}, \tag{B.9}$$

$$\sigma_D^2 = Var\left(\tilde{e}_D | \mathbf{F1}_{D-1}\right). \tag{B.10}$$

With the common knowledge assumption the demand of the domestic speculator (B.7) reads:

$${}^f W_{D-1}^{kf} = \frac{g_D^f}{b^k\, \sigma_D^2\, R_{D-1}^f}. \tag{B.11}$$

This equation simply states that the amount of risky asset held by speculator k during one day is proportional to the expected gain g_D and inversely proportional to its relative

According to the assumptions 1 to 5, *the wealth invested* in *foreign currency by the whole set of domestic speculators* reads by using equation (B.11):

$$^{f}\mathbf{W}_{D-1}^{df} = \sum_{k \in SD_{D-1}} {}^{f}W_{D-1}^{kf} = \sum_{k \in SD_{D-1}} \frac{g_D}{b^k \, \sigma_D^2 \, R_{D-1}^f},$$

where SD_{D-1} is the set of domestic speculators during D day, i. e. $\{(D-1,\ D)\}$. The *expected demand for foreign currency by domestic speculators* during D day thus reads:

$$\mathbf{SPE}_D^{df} = {}^{f}\mathbf{W}_{D-1}^{df} - {}^{f}\mathbf{W}_{D-2}^{df} = \sum_{k \in SD_{D-1}} {}^{f}W_{D-1}^{kf} - \sum_{k \in SD_{D-2}} {}^{f}W_{D-2}^{kf},$$

$$\mathbf{SPE}_D^{df} = \sum_{k \in SD_{D-1}} \left(\frac{g_D^f}{b^k \, \sigma_D^2 \, R_{D-1}^f} \right) - \sum_{k \in SD_{D-2}} \left(\frac{g_{D-1}^f}{b^k \, \sigma_{D-1}^2 \, R_{D-2}^f} \right). \quad \text{(B.12)}$$

Behaviour of the foreign speculator

The foreign speculator maximises the conditional expectation of its wealth denominated in its "domestic" currency which is the foreign currency f in the domestic market under investigation. Its risky asset is the domestic currency d. His risk aversion is $b_f{}^k$. The optimising problem of the foreign speculator is deduced from the domestic one by substituting the exchange rates by their inverses and his demand for domestic currency, is equal to the opposite demand for foreign currency denominated in domestic currency. His budget constraint reads:

$$\tilde{W}_D^{kf} = W_{D-1}^{kf} \, R_{D-1}^f + {}^{d}W_{D-1}^{kd} \left(R_{D-1}^d \, \frac{1}{\tilde{e}_D} - R_{D-1}^f \, \frac{1}{e_{D-1}} \right).$$

For analytical tractability in the determination of the optimal behaviour of the foreign speculator, a linearization around the conditional expectation of the forthcoming exchange rate. By using equation (B.6), the wealth, denominated in foreign currency, of foreign speculator k reads:

$$\tilde{W}_D^{kf} = W_{D-1}^{kf} \, R_{D-1}^f + \frac{R_{D-1}^f \, g_D^{kf}}{E\left[\tilde{e}_D \middle| \mathbf{F}_{D-1}^k\right]} \, {}^{f}W_{D-1}^{kf} + \frac{R_{D-1}^f}{E\left[\tilde{e}_D \middle| \mathbf{F}_{D-1}^k\right]} \, {}^{f}W_{D-1}^{kf} \, \tilde{\varepsilon}_D.$$

The first order condition of the optimising problem of the foreign speculator gives his/her demand for foreign currency:

$$^{f}W_{D-1}^{kf} = \frac{E\left[\tilde{e}_D \middle| \mathbf{F}_{D-1}^k\right]}{b_f{}^k} \, \frac{g_D^{kf}}{(\sigma_D^k)^2 \, R_{D-1}^f},$$

which states that the wealth in foreign currency held by the foreign speculator is identical to the domestic speculator's wealth after dividing the trader's risk aversion by the conditional expectation of the exchange rate at the forthcoming fixing (compare to equation (B.7)).

For analytical tractability the following assumption is made:

Assumption B.6.

The foreign speculator's risk aversion is constant in domestic currency unit.

$$b^k = \frac{b_f{}^k}{E\left[\tilde{e}_D \middle| \mathbf{F}_{D-1}^k\right]}.$$

Consequently, whatever his/her nationality the speculator's wealth, denominated in foreign currency, is given by equation (B.11).

The wealth invested in foreign currency by foreign speculators is:

$$^f\mathbf{W}_D^{ff} = \sum_{k \in SF_{D-1}} {}^fW_D^{kf} = \sum_{k \in SF_{D-1}} \frac{g_D^f}{b^k\, \sigma_D^2\, R_{D-1}^f}$$

where SF_{D-1} is the set of foreign speculators during D day.

The *expected demand for foreign currency by foreign speculators* during D day thus reads:

$$\mathbf{SPE}_D^{ff} = {}^f\mathbf{W}_{D-1}^{ff} - {}^f\mathbf{W}_{D-2}^{ff} = \sum_{k \in SF_{D-1}} {}^fW_{D-1}^{kf} - \sum_{k \in SF_{D-2}} {}^fW_{D-2}^{kf},$$

$$\mathbf{SPE}_D^{ff} = \sum_{k \in SF_{D-1}} \left(\frac{g_D^f}{b^k\, \sigma_D^2\, R_{D-1}^f}\right) - \sum_{k \in SF_{D-2}} \left(\frac{g_{D-1}^f}{b^k\, \sigma_{D-1}^2\, R_{D-2}^f}\right). \qquad \text{(B.13)}$$

Total expected demand for foreign currency

According to the previous assumptions, the total expected demand for foreign currency during D day is the sum of the sets of domestic and foreign speculators, equations (B.12) and (B.13):

$$S\tilde{P}E_D^f{}^* = \sum_{k \in ST_{D-1}} \frac{g_D^f}{b\, \sigma_D^2\, R_{D-1}^f} - \sum_{k \in ST_{D-2}} \frac{g_{D-1}^f}{b\, \sigma_{D-1}^2\, R_{D-2}^f}, \qquad \text{(B.14)}$$

$$S\tilde{P}E_D^f{}^* = \frac{g_D^f}{b\, \sigma_D^2\, R_{D-1}^f} NS_{D-1} - \frac{g_{D-1}^f}{b\, \sigma_{D-1}^2\, R_{D-2}^f} NS_{D-2}, \qquad \text{(B.15)}$$

where NS_{D-1} is the total number of speculators in the set ST_{D-1}:

$$SD_{D-1} \cup SF_{D-1} = ST_{D-1}$$

and b is the harmonic mean of the speculators' risk aversion:

$$\frac{NS_{D-1}}{b} = \sum_{k \in ST_{D-1}} \frac{1}{b^k}.$$

APPENDIX C[1]

1. *Optimal behaviour of a domestic bank when it is a "price maker"*

At a given time t within the intrafixing period[2] $\{(D-1,\ D)\}$, as a "price maker", the bank i is quoting and its quote is part of its information set, i. e. $t \in \tau^i$ and $e_t \notin \Phi_t^i$. Because at all time t, within the intrafixing period $\{(D-1,\ D)\}$, the quote of the quoting bank is the OTC market exchange rate prevailing at that time, the bank's label is omitted, to notify this value is a market one.

In its optimizing problem the case $k = 0$ has to be separately considered since at that time the bank's demand is deterministic. The bank's optimizing problem reads:

$$\underset{\left\{d^{if}_{t+k\notin\tau^i}\right\},\ \left\{e_{t+k\in\tau^i}\right\}}{Max}\ E\left[\widetilde{W}_D^{id}\middle|\Phi_t^i\right]$$

subject to:[3]

$$\widetilde{W}_D^{id} = W_t^{id} + \left({}^fW_t^{if} + Q_t^{if} + TOI_t^{if}\right)\left(\widetilde{e}_D - e_t\right) + \sum_{\substack{k=1\\(t+k\notin\tau^i)}}^{BP-t}\left(\widetilde{d}^{if}_{t+k} + TOI^{if}_{t+k}\right)\left(\widetilde{e}_D - \widetilde{e}_{t+k}\right)$$

$$+ \sum_{\substack{k=1\\(t+k\in\tau^i)}}^{BP-t}\left(\widetilde{Q}^{if}_{t+k} + TOI^{if}_{t+k}\right)\left(\widetilde{e}_D - \widetilde{e}_{t+k}\right)$$

with:

$$W_t^{id} = {}^dW_{t-1}^{id} + {}^fW_{t-1}^{if}\ e_t$$

$${}^fW_t^{if} = {}^fW_{t-1}^{if} + \left(d_t^{if} + TOI_t^{if}\right) \qquad \textit{if the bank is not quoting at } t > \tau_l^i,$$

$${}^fW_t^{if} = {}^fW_{t-1}^{if} + \left(\widetilde{Q}_t^{if} + TOI_t^{if}\right) \qquad \textit{if the bank is quoting at } t = \tau_l^i,$$

where:

W_t^{id} = total wealth at time t denominated in domestic currency d,

${}^dW_{t-1}^{kd}$ = wealth of bank i held, at time $t-1$, in domestic currency and denominated in domestic currency,

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[1] When necessary look to the main text for the definitions of symbols.

[2] The short hand notation $\{(D-1,\ D)\}$, instead of $(D-1,\ D)\cap\mathbf{N}$, is chosen for the sake of simplicity.

[3] The symbols in the sums of the RHS of the budget constraint means: (i) in the first sum in the RHS $t+k \notin \tau^i$, $k \geq 1$ denotes the times at which the bank is not quoting (it is a "price taker"); (ii) in the second sum in the RHS $t+k \in \tau^i$, $k \geq 1$ denotes the values taken by the times at which the bank is quoting (it is a "price maker").

${}^{d}W_{t-1}^{kd}$ = wealth of bank i held, at time $t-1$, in domestic currency and denominated in domestic currency,

${}^{f}W_{t-1}^{kf}$ = wealth of bank i held, at time $t-1$, in foreign currency f and denominated in foreign currency,

$\tilde{d}_{t+k}^{if}$ = bank's demand for foreign currency,

$\tilde{e}_D$ = exchange rate at the forthcoming fixing D,

$\tilde{e}_{t+k}$ = exchange rate at the forthcoming intraday times at which the bank is quoting,

TOI_{t+k}^{if} = total of commercial and speculative orders,

$\tilde{Q}_t^{if}$ = the transactions the bank i has to accept when it is quoting, i. e. the sum of the other banks' demand:

$$\tilde{Q}_t^{if} = -\sum_{j \neq i=1}^{B} \tilde{s}_t^{jf}, \qquad \text{(C.1)}$$

$\tilde{s}_t^{jf}$ is given by equation (C.3) and τ_t^i is its last quoting time.

Two cases have to be considered:

(1) *The first order conditions with respect to the bank's demands in the future* $\left\{d_{t+k}^{if}\right\}$ is the same as in the "price taker" $k \geq 1$ case; they are not taken into account.

(2) *The first order condition with respect to the bank's quotes* $\tilde{e}_{t+k}$, $t+k \in \tau^i$. The demand to the quoting bank i is given by equation (C.1) and is a function of its quote: $\tilde{Q}_{t+k}^{if} = \tilde{Q}_{t+k}^{if}\left(\tilde{e}_{t+k}\right)$.

The first order conditions are for $t+k \in \tau^i$:

(a) *For* $k = 0$, with $Q_t^{if} = Q_t^{if}\left(e_t\right)$:

$$\frac{\partial E\left[\tilde{W}_D^{id} \middle| \Phi_t^i\right]}{\partial e_t} = -{}^{f}W_t^{if} - TOI_t^{if} + \frac{\partial}{\partial e_t} E\left[Q_t^{if}\left(\tilde{e}_D - e_t\right) \middle| \mathbf{F}_t^i\right] = 0.$$

$${}^{f}W_t^{if} + TOI_t^{if} = E\left[\tilde{e}_D \frac{\partial Q_t^{if}}{\partial e_t} \middle| \Phi_t^i\right] - e_t\, E\left[\frac{\partial Q_t^{if}}{\partial e_t} \middle| \Phi_t^i\right] - E\left[Q_t^{if} \middle| \Phi_t^i\right].$$

Assuming the independence of the two terms involved in the first conditional expectation in the right hand side of the previous equation, the first order condition becomes:

$$e_t = E\left[\tilde{e}_D \middle| \Phi_t^i\right] - \frac{1}{\varepsilon}\left(1 + \frac{{}^{f}W_t^{if} + TOI_t^{if}}{E\left[Q_t^{if} \middle| \Phi_t^i\right]}\right)$$

where

$$\varepsilon = \frac{E\left[\frac{\partial Q_t^{if}}{\partial e_t}\middle|\Phi_t^i\right]}{E\left[Q_t^{if}\middle|\Phi_t^i\right]}$$

is the elasticity of the expected demand to the bank.

Determination of the elasticity of the total demand the quoting bank is facing.

At a given time t within the $]D-1, D[$ intrafixing period, bank i is quoting e_t. Let $e_t^j = E\left[\tilde{e}_D\middle|\Phi_t^j\right]$ the conditinal expectations of the forthcoming fixing by bank $j \neq i$, $F\left(e_t^j\right)$ their distribution function and $f\left(e_t^j\right)$ their probability density function.

(1) Banks such that their conditional expectations is greater than the quote e_t buy foreign currency to bank i. The proportion of banks which buy is:

$$\int_{e_t}^{\infty} f\left(e_t^j\right)\, de_t^j = 1 - F\left(e_t\right).$$

(2) Banks such that their conditional expectations is smaller than the quote e_t sell foreign currency to bank i. The proportion of banks which sell is:

$$\int_{-\infty}^{e_t} f\left(e_t^j\right)\, de_t^j = F\left(e_t\right).$$

Since each transaction is $q^f *$, then the total demand to bank i, from the $(N - 1)$ other banks, reads (difference between its sales and purchases):

$$Q_t^i\left(e_t\right) = (N-1)\,\left[1 - F\left(e_t\right)\right] q^f * - (N-1)\; F\left(e_t\right) q^f *$$
$$Q_t^i\left(e_t\right) = (N-1)\,\left[1 - 2\,F\left(e_t\right)\right] q^f *.$$

The effect of a change in the quote on the total demand is:

$$\frac{\partial Q_t^i\left(e_t\right)}{\partial e_t} = -2(N-1)\; f\left(e_t\right) q^f *.$$

The more centered the distribution of the conditional expectations $E\left[\tilde{e}_D\middle|\Phi_t^j\right]$ around the quote, the steeper the (absolute value of) slope of the total demand.

Assuming the distribution of the banks with respect to their expectations has a sufficiently small standard deviation, then the slope of the total demand is large enough with respect to the total demand, i. e. $\frac{\partial \tilde{Q}_t^{if}\left(e_t\right)}{\partial e_t} >> \tilde{Q}_t^{if}\left(e_t\right)$, making the elasticity ε of the total demand very large. For exemple, if banks have all the same expectations the density of probability function is a δ-function, then the total demand is an Heaviside function.

Consequently, taking the infinite limit for the elasticity of the expected demand the bank's quote reads:

$$e_t = E\left[\tilde{e}_D\middle|\Phi_t^i\right]. \qquad \text{(C.2)}$$

(b) *For* $k \geq 1$.

The solutions are analogous to the $k = 0$ case by substituting for e_t by $E\left[\tilde{e}_{t+k} \middle| \Phi_t^i\right]$. Since these expected exchange rates do not intervene at time t, they are not taken into account.

2. *Optimal behaviour of a domestic bank when it is a "price taker"*

At a given time t within the intrafixing period $\{(D-1,\, D)\}$, as a "price taker", the bank i is not quoting and the quote given by another bank is part of its information set, i. e. $t \notin \tau^i$ and $e_t \in \Phi_t^i$. Because at all time t, within the intrafixing period $\{(D-1,\, D)\}$, the quote of the quoting bank is the OTC market exchange rate prevailing at that time, the bank's label is omitted, to notify this value is a market one.

In its optimizing problem the case $k = 0$ has to be separetely considered since at that time the bank's demand is deterministic. It reads:

$$\underset{\left\{d_{t+k \notin \tau^i}^{if}\right\},\ \left\{e_{t+k \in \tau^i}\right\}}{Max}\ E\left[\tilde{W}_D^{id} \middle| \Phi_t^i\right]$$

subject to[4] :

$$\tilde{W}_D^{id} = W_t^{id} + \left({}^f W_t^{if} + d_t^{if} + TOI_t^{if}\right)\left(\tilde{e}_D - e_t\right) + \sum_{\substack{k=1 \\ (t+k \notin \tau^i)}}^{BP-t} \left(\tilde{d}_{t+k}^{if} + TOI_{t+k}^{if}\right)\left(\tilde{e}_D - \tilde{e}_{t+k}\right)$$

$$+ \sum_{\substack{k=1 \\ (t+k \in \tau^i)}}^{BP-t} \left(\tilde{Q}_{t+k}^{if} + TOI_{t+k}^{if}\right)\left(\tilde{e}_D - \tilde{e}_{t+k}\right)$$

with:

$$W_t^{id} = {}^d W_{t-1}^{id} + {}^f W_{t-1}^{if}\ e_t$$

$${}^f W_t^{if} = {}^f W_{t-1}^{if} + \left(d_t^{if} + TOI_t^{if}\right) \qquad \textit{if the bank is not quoting at } t > \tau_1^i,$$

$${}^f W_t^{if} = {}^f W_{t-1}^{if} + \left(\tilde{Q}_t^{if} + TOI_t^{if}\right) \qquad \textit{if the bank is quoting at } t = \tau_1^i,$$

where:

W_t^{id} = total wealth at time t denominated in domestic currency d,

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[4] The symbols in the sums of the RHS of the budget constraint means: (i) in the first sum in the RHS $t + k \notin \tau^i$, $k \geq 1$ denotes the values taken by the variables only at times at which the bank is not quoting (it is a "price taker"); (ii) in the second sum in the RHS $t + k \in \tau^i$, $k \geq 1$ denotes the values taken by the variables at only times at which the bank is quoting (it is a "price maker").

${}^{d}W_{t-1}^{kd}$ = wealth of bank i held, at time $t-1$, in domestic currency and denominated in domestic currency,

${}^{f}W_{t-1}^{kf}$ = wealth of bank i held, at time $t-1$, in foreign currency f and denominated in foreign currency,

$\tilde{d}_{t+k}^{if}$ = bank's demand for foreign currency,

$\tilde{e}_D$ = exchange rate at the forthcoming fixing D,

$\tilde{e}_{t+k}$ = exchange rate at the forthcoming intraday times at which the bank is quoting,

TOI_{t+k}^{if} = total of commercial and speculative orders,

$\tilde{Q}_t^{if}$ = the transactions the bank i has to accept when it is quoting, i. e. the sum of the other banks' demand:

$$\tilde{Q}_t^{if} = -\sum_{j \neq i = 1}^{B} \tilde{s}_t^{jf},$$

$\tilde{s}_t^{jf}$ is given by equation (C.3) and τ_t^i is its last quoting time.

For all the times from t to the fixing D, i. e. $t+k \notin \tau^i$, the bank's demands for foreign currency d_{t+k}^{if} and quotes $\tilde{e}_{t+k}$ in the future are given by the first order conditions. Three conditions have to be considered:

(1) *First order condition with respect to* $\left\{d_{t+k}^{if}\right\}$.

(a) *For k = 0:*

If $E\left[\tilde{e}_D \middle| \Phi_t^i\right] = e_t$ the solution is indetermined and because of costs the demand for currency is set to zero: $d_t^{if} = 0$.

If $E\left[\tilde{e}_D \middle| \Phi_t^i\right] > e_t$, the bank anticipates an appreciation of the foreign currency; its demand reads:

$$d_t^{if} = q^f *$$

and the change in the bank's in foreign currency is:

$$s_t^{if} = q^f * + TOI_t^{if}$$

where $q^f *$ is the bound of the transactions made between two banks.

If $E\left[\tilde{e}_D \middle| \Phi_t^i\right] < e_t$, the bank anticipates a depreciation of the foreign curency; its demand reads:

$$d_t^{if} = -q^f *$$

and the change in the bank's in foreign currency is:

$$s_t^{if} = -q^f * + TOI_t^{if}. \tag{C.3}$$

To end up, at time t within the $]D-1, D[$ intrafixing period the bank's demand for foreign currency, reads:

$$d_t^{if} = Sign\left\{E\left[\tilde{e}_D \middle| \mathbf{F}_t^i\right] - e_t\right\} q^f *$$

and its change in position in foreign currency is:

$$s_t^{if} = d_t^{if} + TOI_t^{if}.$$

(b) *For* $k \geq 1$.

The solutions are analogous to the $k = 0$ case by substituting for e_t by $E\left[\tilde{e}_{t+k} \middle| \Phi_t^i\right]$.

Since these demands do not intervene at time t, they are not taken into account.

(2) *The first order condition with respect to the bank's quotes in the future* $\tilde{e}_{t+k}$, $t+k \in \tau^i$, leads to:

$$\tilde{e}_{t+k} = E\left[\tilde{e}_D \middle| \Phi_t^i\right] - \frac{1}{\varepsilon}\left(1 + \frac{TOI_t^{if}}{E\left[Q_t^{if} \middle| \Phi_t^i\right]}\right) \quad \text{for } t+k \in \tau^i$$

where ε is the elasticity of the expected demand to the bank which quotes. According to the functioning of the market, this elasticity can be set to infinity (see proof in § 1, equation (C.2): case where the bank is a "price maker"), then the bank' quote reads:

$$\tilde{e}_{t+k} = E\left[\tilde{e}_D \middle| \Phi_t^i\right] \quad \text{for } t+k \in \tau^i.$$

Remark: $\tilde{e}_{t+k}$ is k-independent. The previous equation is a non arbitrage equation of the martingale type; whatever is the future, it is determine by the available information at a given time t.

3. *Optimal behaviour of a foreign bank*

The foreign bank maximizes the conditional expectation of its wealth denominated in its "domestic" currency which is the foreign currency f in the domestic market under investigation. Its risky asset is the domestic currency d. Since the optimal quote and demand for the foreign currency f of the domestic bank, given by propositions 1 and 2 in the main text, depends only on the characteristics of the domestic bank and the bounded transaction, the optimal behaviour of the foreign bank is determined by an equivalent optimizing problem. The optimizing problem of the foreign bank is deduced from the domestic one by substituting the exchange rates by their inverses and its demand for domestic currency, is equal to the opposite demand for foreign currency denominated in domestic currency. The optimal quote and demand for domestic currency are given by the following propositions:

PROPOSITION 1.a: *When the bank is a "price maker", its optimal quote is:*[5]

$$\frac{1}{e_t} = E\left[\frac{1}{\tilde{e}_D}\middle|\Phi_t^i\right]$$

and the change in the bank's position in foreign currency is:

$$s_t^{if} = \tilde{Q}_t^{if} + TOI_t^{if}.$$

Proof: By a calculation similar to the one presented in the case of the domestic bank in § 1.

PROPOSITION 2.a: *When the bank is a "price taker", its optimal demand for the foreign currency, is:*

$$d_t^{if} = -Sign\left\{E\left[\frac{1}{\tilde{e}_D}\middle|\Phi_t^i\right] - \frac{1}{e_t}\right\} q^f *$$

and the change in the bank's position in foreign currency is:

$$s_t^{if} = d_t^{if} + TOI_t^{if}.$$

Proof: By a calculation similar to the one presented in the case of the domestic bank in § 1.

For analytical tractability, the optimal behaviour of the foreign bank is determined by a linearization of its optimal problem.

By linearizing around an exchange rate $\bar{e}$, i. e. the long-run exchange rate, the wealth, denominated in foreign currency, of foreign bank *i* reads:

$$\bar{e}\,\tilde{W}_D^{if} = \bar{e}\,W_t^{if} + \frac{{}^d W_t^{id}}{e_t}\left(e_t - \tilde{e}_D\right) + \sum_{t+k \notin \tau^i} \frac{\tilde{d}_{t+k}^{id} + TOI_{t+k}^{id}}{\tilde{e}_{t+k}}\left(\tilde{e}_{t+k} - \tilde{e}_D\right)$$

$$+ \sum_{t+k \in \tau^i} \frac{\tilde{Q}_{t+k}^{id} + TOI_{t+k}^{id}}{\tilde{e}_{t+k}}\left(\tilde{e}_{t+k} - \tilde{e}_D\right).$$

The wealth, denominated in domestic currency, of foreign bank *i* becomes:

$$\tilde{W}_D^{id} = W_t^{id} + {}^f W_{t-1}^{if}\left(\tilde{e}_D - e_t\right) + \sum_{t+k \notin \tau^i}\left(\tilde{d}_{t+k}^{if} + TOI_{t+k}^{if}\right)\left(\tilde{e}_D - \tilde{e}_{t+k}\right)$$

$$+ \sum_{t+k \in \tau^i}\left(\tilde{Q}_{t+k}^{if} + TOI_{t+k}^{if}\right)\left(\tilde{e}_D - \tilde{e}_{t+k}\right).$$

This budget constraint is identical to the domestic bank's budget constraint given by equation (10b) in the main text. Since, on the one hand, domestic and foreign banks have the same budget constraint and on the other hand,

[5] Recall that because at all time *t*, within the intrafixing period $]D-1, D[$, which is a shorthand notation of the D^{th} day OTC market, the quote of the quoting bank is the OTC market exchange rate prevailing at that time, the bank's label is omitted, to notify this value is a market one.

$$Max\, E\left[\tilde{W}_D^{if}\middle|\Phi_t^i\right] \equiv Max\, E\left[\frac{\tilde{W}_D^{id}}{\bar{e}}\middle|\Phi_t^i\right] \equiv Max\, E\left[\tilde{W}_D^{id}\middle|\Phi_t^i\right] \text{ since } \bar{e} > 0,$$

then the optimal quote and demand for foreign currency of a foreign bank is given, after linearization, by propositions 2 and 3 in the main text.

APPENDIX D[1]

The model of the fixing

A. Commercial banks

At the fixing time, bank i's transactions reduce to its net purchases FIX_D^{if}. Let ΔPOS_D^{if} be the change in the position in the foreign currency of the bank during the OTC market is opened which is the sum of its transactions with the other banks and with its clients. During the D^{th} day the change in the position in the foreign currency of bank i, $DPOS_D^{if}$, is the sum of ΔPOS_D^{if}, and of its net purchases FIX_D^{if}. We now assume:

Assumption D.1:

Every bank clears its position in foreign currency, at the fixing.

$$DPOS_D^{if} = \Delta POS_D^{if} + FIX_D^{if} = 0.$$

Proposition D.1: *The banks' demand at the fixing is equal to the opposite of the total of the orders their clients' demand during the opening of the OTC market:*

$$\sum_{i=1}^{B} FIX_D^{if} = -\sum_{i=1}^{B} \sum_{t=1}^{BP} TOI_t^{if} .$$

Of course, these orders are the sum of commercial and speculative orders; hence:

$$\sum_{i=1}^{B} F\tilde{I}X_D^{if} = -\sum_{i=1}^{B} T\tilde{O}I_D^{if} = -\sum_{i=1}^{B} \sum_{t=1}^{BP} \left(\tilde{X}_t^{id} / \tilde{e}_D - \tilde{M}_t^{if} - S\tilde{P}E_t^{if} \right)$$

which, alternatively can be written as the sum of the daily trade balance and the total speculative demand:

$$\sum_{i=1}^{B} F\tilde{I}X_D^{if} = -\left(\tilde{X}_D^{f} - \tilde{M}_D^{f} \right) + S\tilde{P}E *_D^{f}. \qquad \text{(D.9)}$$

♦ The proof is just a matter of accounting, as following:

Let ΔPOS_D^{if} be the change in the position in the foreign currency of bank i during the OTC market is opened, i. e. the period[2] $\{(D-1,\ D)\}$. It is the sum of its transactions with the other banks and with its clients:

(i) When bank i is quoting, at a given time $t \in \tau^i$, the change in the bank's position in foreign currency is given by equations (C.1) and (C.3) of appendix C:

$$s_t^{if} = \tilde{Q}_t^{if} + TOI_t^{if} = -\sum_{j \neq i=1}^{B} d_t^{if} + TOI_t^{if}, \quad t \in \tau^i. \qquad \text{(D.1)}$$

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[1] When necessary look to the main text for the definitions of symbols.

[2] The short hand notation $\{(D-1,\ D)\}$, instead of $(D-1,\ D) \cap \mathbf{N}$, is chosen for the sake of simplicity.

(ii) When bank i is not quoting, at a given time $t \notin \tau^i$, the change in the bank's position in foreign currency is given by equation (C.3) of appendix C:

$$s_t^{if} = d_t^{if} + TOI_t^{if}, \quad t \notin \tau^i. \tag{D.2}$$

Over the period $\{(D-1,\ D)\}$, the change in the bank's position ΔPOS_D^{if} is the sum of the changes in the positions given by equations (D.1) and (D.2); hence:

$$\Delta POS_D^{if} = \sum_{t \notin \tau^i} \left\{ d_t^{if} + TOI_t^{if} \right\} + \sum_{t \in \tau^i} \left\{ -\sum_{j \neq i=1}^{B} d_t^{jf} + TOI_t^{if} \right\}. \tag{D.3}$$

Consequently, during the D^{th} day, the change in the position in the foreign currency of bank i, $DPOS_D^{if}$, is the sum of ΔPOS_D^{if}, equation (D.3), its net purchases FIX_D^{if}. Hence:

$$DPOS_D^{if} = \Delta POS_D^{if} + FIX_D^{if} + TOF_D^{if}. \tag{D.4}$$

At the fixing, because of assumption D.1, the demand of bank i is, thus:

$$FIX_D^{if} = -\Delta POS_D^{if}.$$

Summing over the B banks, the net aggregate demand for the foreign currency reads:

$$\sum_{i=1}^{B} FIX_D^{if} = -\sum_{i=1}^{B} \Delta POS_D^{if}. \tag{D.5}$$

By using equation (D.3), the demand to bank i at the fixing, equation (D.5) reads:

$$\sum_{i=1}^{B} FIX_D^{if} = -\sum_{i=1}^{B} \sum_{t \notin \tau^i} d_t^{if} + \sum_{i=1}^{B} \sum_{t \in \tau^i} \sum_{j \neq i} d_t^{jf} - \sum_{i=1}^{B} TOI_D^{if}$$

which reduces to:

$$\sum_{i=1}^{B} FIX_D^{if} = -\sum_{i=1}^{B} TOI_D^{if} \tag{D.6}$$

since:

$$-\sum_{i=1}^{B} \sum_{t \notin \tau^i} d_t^{if} + \sum_{i=1}^{B} \sum_{t \in \tau^i} \sum_{j \neq i} d_t^{jf} = 0.$$

Equation (D.6) shows that, if all the banks clear their position at the fixing, their net demand is the opposite of the total of the clients' orders executed in the OTC market and at the fixing.

The orders are splitted into commercial orders and speculative orders executed in the OTC market, equation (15) of the main text, respectively, TCI and TSI in the OTC market:

$$TOI_D^{if} = TCI_D^{if} + TSI_D^{if}, \tag{D.7}$$

where

$$TCI_D^{if} = \sum_{t=1}^{BP} -\tilde{M}_t^{if} + \frac{\tilde{X}_t^{id}}{e_t}, \tag{D.8a}$$

$$TSI_D^{if} = \sum_{t=1}^{BP} -SP\tilde{E}_t^{if} \,. \qquad \text{(D.8b)}$$

Hence by using equations (15) in the main text, (D.7) to (D.8b), the total of the banks' net purchases, equation (D.5), becomes:

$$\sum_{i=1}^{B} FIX_D^{if} = -\sum_{i=1}^{B}\left(TCI_D^{if} + TSI_D^{if}\right) = -\sum_{i=1}^{B} TCI_D^{if} - \sum_{i=1}^{B} TSI_D^{if} \,,$$

which becomes by introducing the daily trade balance $(\tilde{X}_D^f - \tilde{M}_D^f)$ and the total speculative demand $SPE_D^f *$:

$$\sum_{i=1}^{B} F\tilde{I}X_D^{if} = -\left(\tilde{X}_D^f - \tilde{M}_D^f\right) + S\tilde{P}E *_D^f. \qquad \text{(D.9)}$$

Q.E.D. ♦

B. Central Banks, Speculators and Market Equilibrium

It has already been pointed out that, for the sake of simplicity, the purchases and sales of the central banks on the market are neglected; hence we assume:

Assumption D.2:
Foreign exchange reserves are held constant.

With such an assumption, the equilibrium of the exchange market at the fixing time D reads from equation (D.9):

$$\sum_{i=1}^{B} F\tilde{I}X_D^{if} = -\left(\tilde{X}_D^f - \tilde{M}_D^f\right) + S\tilde{P}E_D^f = 0. \qquad \text{(D.10)}$$

The previous equilibrium condition (D.10) becomes with some algebra calculations using equations (7) and (8a) of the main text, and equation (B.15) of appendix B:

$$S\tilde{P}E *_D^f = u *_D + \tilde{v}_D - \delta_D \, \tilde{e}_D \qquad \text{(D.11)}$$

where:

$$u *_D = \frac{2\,\lambda_D\, X_M^d}{e_{M-1}} - \lambda_D\, M_M^f = \lambda_D \left(\frac{2\,X_M^d}{e_{M-1}} - M_M^f\right) \qquad \text{(D.12a)}$$

$$\tilde{v}_D = \frac{\tilde{v}_D^x}{e_{M-1}} - \tilde{v}_D^m - \tilde{v}_D^s, \qquad \text{(D.12b)}$$

and the speculators' expected demand for currency $S\tilde{P}E *_D^f$ the formal expression of which has been determined in appendix B, equation B.15) has to be calculated now.

For the sake of simplicity, the following usual assumptions are made:

Assumption D.3:

The number of speculators is constant, i. e.:

$$NS_{D-1} = NS_{D-2} = NS. \qquad \forall\ D \geq 3 \qquad \text{(D.13)}$$

Of course, central banks also influence the exchange rate through their monetary policy which, for "small" countries, is generally assumed to be a reaction function linking the level of the short domestic interest rate to the values of variables characterising domestic and foreign conjunctures. On the very short period, the evolution of the exchange rate depends on the domestic and foreign interest rates and on the value of the trade balance. A question should now be answered: how the domestic central bank will fix its short term interest rate? It is shown in Chauveau (1990)[3] that, under fairly standard assumptions[4], the optimal monetary policy may be written:

$$r^d = r^f + \pi^d - \pi^f + \sum_{k=1}^{\infty}\left(\varsigma_k . (s(-k) - s*\right),$$

where r^d (r^f) is the domestic (foreign) interest rate π^d (π^f) is the trend value of domestic (foreign) inflation rate and s ($s*$) is the current value (long run equilibrium value) of the real exchange rate. However, such an analysis is relevant on a quarterly or, at most, on a monthly one as it requires that the variables are observed simultaneously. This is no more the case, if we take into account daily decisions but the theoretical result obtained on a quarterly basis -the existence of unitary coefficients for the foreign interest rate and the inflation rate- suggests the following assumption:

Assumption D.4:

On the very short run, the central bank keeps constant the difference between the foreign and the domestic interest rate, i. e.:

$$\rho_D = \rho. \qquad \forall\ D \geq 3 \qquad \text{(D.14)}$$

This is a realistic although very elementary behaviour of the central bank; the gap between the two rates is modified but when news about domestic or foreign inflation become available or when discretionary policies are undertaken.

Assumption D.5:

Assume the monthly exports and imports are uniformally distributed over days, i. e.:

$$\lambda_D = \lambda \qquad \forall\ D \geq 3. \qquad \text{(D.15)}$$

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[3] Chauveau, T. (1990). "Optimal Monetary Policies in a Small Open Economy", in *Monetary Policy*, Artus, P. and Barroux, Y., eds, Amsterdam: Kluwer Academic Press.

[4] The short term domestic interest rate is the instrument used for monetary policy, no intermediary target is selected, and the central banks want to minimise the variance of the rate of growth of the exchange rate.

Then the definition of δ_D, given by equation (8d) in the main text, becomes:

$$\delta_D = \delta = \frac{\lambda\, X_M^d}{e_{M-1}^2}. \tag{D.16}$$

Under the assumptions D.3 to D.5 and neglecting second order terms in interest rates $\left(R^f \approx 1\right)$, and using the relationship (B.15) of appendix B for the speculator's expected demand for currency, the equilibrium condition (D.11) of the forex market at the fixing becomes:

$$E\left[\tilde{e}_D \left| \mathbf{F1}_{D-1}\right.\right] - \rho\, \tilde{e}_{D-1} - E\left[\tilde{e}_{D-1} \left| \mathbf{F1}_{D-2}\right.\right] + \rho\, \tilde{e}_{D-2} = \beta\, \left(u^* + \tilde{\eta}_D - \delta\, \tilde{e}_D\right) \tag{D.17}$$

with:

$$\beta = b\, \psi_0{}^2\, \sigma^2\, R^f / NS. \tag{D.18}$$

This equation is analogous to the one obtained by Broze, Gourieroux and Szafarz (1989, equation 8, p. 104) in the framework of a rational expectations model for the price of a financial asset when the supply is noisy. Then the exchange rate $\tilde{e}_D$ obeys the following second order stochastic difference equation:

$$c\, \tilde{e}_D - \tilde{\varepsilon}_D - \chi\, \tilde{e}_{D-1} + \tilde{\varepsilon}_{D-1} + \rho\, \tilde{e}_{D-2} = \beta\, \left(u^* + \tilde{v}_D\right) \tag{D.19}$$

where:

$$\tilde{\varepsilon}_D = \tilde{e}_D - E\left[\tilde{e}_D \left| \mathbf{F1}_{D-1}\right.\right] \tag{D.20}$$

$$c = 1 + \beta\, \delta \tag{D.21}$$

$$\chi = 1 + \rho. \tag{D.22}$$

If expectations are rational, ε may be any martingale difference sequence (i.e. as a process orthogonal to the past values of other variables (Broze, Gourieroux and Szafarz, 1989)) . Following these authors, we look for solutions of the form:

$$\tilde{e}_D = \mu_D + \sum_{K=0}^{\infty} \psi_K\, \tilde{v}_{D-K} = \mu_D + P(B)\, \tilde{v}_D \tag{D.23}$$

when:

Assumption D.6:
There is a zero interest rate differential between the two countries.

The equilibrium condition (D.19) is then solved, by using the undetermined coefficients method, in two steps, respectively for the deterministic part and for the stochastic one. When the speculators are risk averse $(\beta > 0)$ and when the daily trade balance depends on the exchange rate $(\delta \neq 0)$ the solution is given by:

(i) To obtain *the stochastic part* of the solution of the equilibrium condition (D.19), one must take into account the following statistical properties of the forecasting error:

$$\tilde{\varepsilon}_D = \psi_0\, \tilde{v}_D \tag{D.25a}$$

$$E[\tilde{\varepsilon}] = 0 \tag{D.25b}$$

$$Var[\tilde{\varepsilon}] = \psi_0^2 \ \sigma_v^2, \tag{D.25c}$$

where σ_v^2 is the variance of the daily noise $\tilde{v}_D$; the stochastic part of equation (D.19) thus reads:

$$c \ P(B) \ \tilde{v}_D - \psi_0 \ \tilde{v}_D - \chi \ \ B \ P(B) \ \tilde{v}_D + \psi_0 \ \ B \ \tilde{v}_D + \rho \ B^2 \ \ P(B) \ \tilde{v}_D = \beta \ \tilde{v}_D \tag{D.26}$$

and its solution is:

$$P(B) = \frac{\psi_0 - \psi_0 \ B\beta}{c - \chi \ \ B + \rho \ B^2}.$$

Explicitly, the ψ's are given by the following equations:

$$\psi_0 = \frac{1}{\delta} \tag{D.27a}$$

$$\psi_1 = \frac{\rho}{(1+\beta \ \delta)\,\delta} \tag{D.27b}$$

$$\psi_K = (1+\beta \ \delta)^{-\frac{K}{2}} \left[\left(\mu_0 - \frac{u^*}{\delta} \right) \cos\theta \ \ K + \frac{\mu_{-1} - \mu_0}{(\beta \ \delta)^{\frac{1}{2}}} \sin\theta \ \ K \right], \ \forall \ K \geq 2. \tag{D.27c}$$

where:

$$\mathrm{tg}\theta = (\beta \ \delta)^{\frac{1}{2}},$$

μ_{-1}, μ_0, are the initial conditions.

Note that the modulus of ψ_K is given by $(1+\beta \ \delta)^{-\frac{K}{2}}$, which according to the possessiveness of the parameters β and δ converges to zero.

(ii) The solution of the *deterministic part* of the equilibrium condition (D.19)

$$c \ \mu_D - \chi \ \ \mu_{D-1} + \rho \ \ \mu_{D-2} = \beta \ \ u^*.$$

reads:

$$\mu_D = \frac{u^*}{\delta} + (1+\beta \ \delta)^{-\frac{D}{2}} \left[\left(\mu_0 - \frac{u^*}{\delta} \right) \cos\theta \ \ D + \frac{\mu_{-1} - \mu_0}{(\beta \ \delta)^{\frac{1}{2}}} \sin\theta \ \ D \right]. \tag{D.28}$$

Note that the modulus of the μ_D is given by $(1+\beta \ \delta)^{-\frac{D}{2}}$, which according to the possessiveness of the parameters β and δ converges to zero. Consequently, μ_D converges to $\frac{u^*}{\delta}$.

All the previous results are put in the following proposition:

PROPOSITION E.2: *Calculated within the framework of the BGS model:*

1. *The value of the exchange rate at the forthcoming fixing reads:*

$$\tilde{e}_D = \mu_D + \sum_{K=0}^{D-1} \psi_K \, \tilde{\nu}_{D-K}. \tag{D.29}$$

2. *Its expectations conditional upon the information held in the past values of the daily noises writes:*

$$E\left[\tilde{e}_D \middle| \mathrm{I}_{D-1}\right] \equiv e*_D = \mu_D + \sum_{K=1}^{D-1} \psi_K \, \tilde{\nu}_{D-K} \tag{D.30}$$

and the fixing exchange rate reads:

$$\tilde{e}_D = e*_D + \psi_0 \, \tilde{\nu}_D. \tag{D.31}$$

3. *The first and second order unconditional moments of* $\tilde{e}_D$ *and its expectation* $e*_D$ *are:*

$$E\left[\tilde{e}_D\right] = E\left[e*_D\right] = \mu_D$$

$$E\left[(e*_D)^2\right] = \mu_D{}^2 + \sigma_\nu{}^2 \left(\sum_{K=1}^{\infty} \psi_K{}^2 \right)$$

$$E\left[(\tilde{e}_D)^2\right] = E\left[(e*_D)^2\right] + \psi_0{}^2 \, \sigma_\nu{}^2$$

$$E\left[\tilde{e}_D \, e*_D\right] = E\left[(e*_D)^2\right]$$

where μ_D *is the deterministic component of the* $\tilde{e}_D$ *and* $\sigma_\nu{}^2$ *the variance of the daily noises.*

The speculators' expected demand for currency being given by equation (B.15) of appendix B, reads:

$$S\tilde{P}E*_D^f = \frac{g_D^f}{b \, \sigma_D^2 \, R_{D-1}^f} NS_{D-1} - \frac{g_{D-1}^f}{b \, \sigma_{D-1}^2 \, R_{D-2}^f} NS_{D-2}.$$

According to the definitions of $\tilde{e}_D$, equation (D.29), and the expectations, equation (D.30), given in the proposition D.2, the speculators' expected demand for currency becomes after a straightforward calculation:

$$\begin{aligned} \beta \; S P\tilde{E}*_D^f &= \mu_D - \chi \; \mu_{D-1} + \rho \; \mu_{D-2} \\ &\quad + \left(\psi_1 - \rho \, \psi_0\right) \tilde{\nu}_{D-1} + \sum_{K=0}^{D-1} \left(\psi_{K+2} - \chi \; \psi_{K+1} + \rho \; \psi_K\right) \tilde{\nu}_{D-1-K} \end{aligned} \tag{D.32}$$

APPENDIX E[1]

The OTC exchange rate dynamics and its conditional statistical properties

A. Each bank observes the noises corresponding to its clients' orders

1. *Determination of the OTC exchange rate*

Formally, the exchange rate at the fixing at D day is the following random variable:

$$\tilde{e}_D : (\Omega, \mathbf{F}, \mathbf{P}) \rightarrow (\Re_+, \mathbf{B}(\Re_+)).$$

At a given time t, within the intrafixing period[2] $\{(D-1,\ D)\}$, when each bank observes the noises corresponding to the orders of its clients, the information structure of the quoting bank $i = \theta(t)$ reads:

$$\mathbf{I}_t^i = \sigma\left(\mathbf{I1}_{D-1} \cup \mathbf{I2}_t^i \cup \mathbf{I3}_t\right) \qquad i = 1 \text{ to } B \text{ and } t = 1 \text{ to } B\ P.$$

Recall that $\mathbf{I1}_{D-1}$ is in fact included in both $\mathbf{I2}_t^i$ and $\mathbf{I3}_t$.

The OTC exchange rate writes:

$$\tilde{e}_t = E\left[\tilde{e}_D \middle| \mathbf{I}_t^i\right].$$

It is the orthogonal projection of $\tilde{e}_D$ on to the closed Hilbert sub-space $H_t^i = L^2\left(\mathbf{F}_t^i\right)$.

As assumed in the main text, the banks determines the model of the fixing from the knowledge of the past values of the exchange rate at the fixing by the BGS' model, which is a linear rational expectations model. This gives time-independent expectations e^*_D within the OTC period:

$$\tilde{e}_D = \mu_D + \sum_{K=0}^{D-1} \psi_K\ \tilde{\nu}_{D-K}\ ;$$

$$e^*_D = \mu_D + \sum_{K=1}^{D-1} \psi_K\ \tilde{\nu}_{D-K}$$

Because e^*_D is a linear functions of the noises relative to the orders, then any observer of the of the past values of the exchange rate as the information given by the noises:

$$\mathbf{F1}_{D-1} = \sigma\left(\tilde{e}_{D-K},\ 1 \le K \le D-1\right) = \sigma\left(\tilde{\nu}_{D-K},\ 1 \le K \le D-1\right) = \mathbf{I1}_{D-1}.$$

$$e^*_D \in H\nu_{D-1}$$

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[1] When necessary look to the main text for the definitions of symbols.

[2] The short hand notation $\{(D-1,\ D)\}$, instead of $(D-1,\ D) \cap \mathbf{N}$, is chosen for the sake of simplicity.

where Hv_{D-1} is the closed Hilbert sub-space of linear functions $\mathbf{I1}_{D-1}$-measurable.

As the second and third source of information, which are respectively the implicit exchange rate and the previous quotes, are variable during the day $\{(D-1,\ D)\}$.

The implicit exchange rate reads:

$$\tilde{e}*_t^i = e*_D + \psi_0 \frac{\bar{\bar{v}}_t^i}{\bar{\lambda}_t^i}.$$

The corresponding information structure is:

$$\sigma\left(\tilde{e}*_t^i\right) = \sigma\left(\mathbf{I1}_{D-1} \cup \sigma\left(\tilde{V}_\tau^i,\ \tau = 1\ to\ t\right)\right) = \mathbf{I2}_t^i.$$

Proposition 5 of the main text gives the following orthogonality condition:

$$E\left[\left(\tilde{e}*_t^i - e*_D\right)e*_D\right] = \frac{\psi_0}{\bar{\lambda}_t^i} E\left[\bar{\bar{v}}_t^i\ e*_D\right] = 0.$$

Let us determine the OTC exchange by considering the banks determination from time $t = 1$.

At time $t = 1$, the quoting bank $\theta(1) = 1$ knows the expectations $e*_D$ and its private information given by the noise $\bar{\bar{v}}_1^1$ corresponding to the orders of its clients. Its information structure reads:

$$\sigma\left(\mathbf{I1}_{D-1} \cup \sigma\left(\bar{\bar{v}}_1^1\right)\right) = \mathbf{I2}_1^1.$$

Its quote reads:

$$\tilde{e}_1 = E\left[\tilde{e}_D \middle| \mathbf{I2}_1^1\right].$$

Because of the orthogonality condition between the expectations $e*_D$ and the noise $\bar{\bar{v}}_1^1$, the two closed Hilbert sub-spaces Hv_{D-1} and Hv_1^1 of linear functions $\mathbf{I1}_{D-1}$-measurable and $\sigma\left(\bar{\bar{v}}_1^1\right)$-measurable are orthogonal. Then the quote, that is to say the OTC exchange rate $\tilde{e}_1$, is a vector of a closed Hilbert sub-space which is the direct sum of these two sub-spaces:

$$H_1^1 = Hv_{D-1} \oplus Hv_1^1.$$

Consequently, the OTC exchange rate $\tilde{e}_1$ is searched as the sum of the expectations $e*_D$ with a vector collinear to the noise $\bar{\bar{v}}_1^1$. The coefficient is determined by minimum of the quadratic error (OLSQ). Then the exchange rate at time $t = 1$ reads:

$$\tilde{e}_1 = e*_D + \psi_0\ \bar{\bar{v}}_1^1.$$

At time $t=2$, the quoting bank $\theta(2)=2$ has three sources of information: the expectations e^*_D, its private information given by the noise $\overline{\overline{v}}_2^2$ corresponding to the orders of its clients and the previous OTC exchange rate $\tilde{e}_1$ gives it the information as the noise $\overline{\overline{v}}_1^1$: $\sigma(\tilde{e}_1)=\sigma(\overline{\overline{v}}_1^1)$. Then its information structure reads:

$$\sigma\left(\mathrm{I}1_{D-1}\cup\sigma\left(\overline{\overline{v}}_1^1\right)\cup\sigma\left(\overline{\overline{v}}_2^2\right)\right)=\mathrm{I}2_2^2.$$

Its quote reads:

$$\tilde{e}_2=E\left[\tilde{e}_D\middle|\mathrm{I}2_2^2\right].$$

Because of the orthogonality condition between, on the one hand, e^*_D and $\overline{\overline{v}}_2^2$, and on the other hand, the two noises $\overline{\overline{v}}_2^2$ and $\overline{\overline{v}}_1^1$, the three closed Hilbert sub-spaces Hv_{D-1}, Hv_2^2 and Hv_1^1 of linear functions respectively $\mathrm{I}1_{D-1}$-measurable, $\sigma\left(\overline{\overline{v}}_2^2\right)$-measurable and $\sigma\left(\overline{\overline{v}}_1^1\right)$-measurable are orthogonal by pair. Then the quote, that is to say the OTC exchange rate $\tilde{e}_2$, is a vector of a closed Hilbert sub-space which is the direct sum of these two sub-spaces:

$$H_2^2=Hv_{D-1}\oplus Hv_2^2\oplus Hv_1^1.$$

Consequently, the quote is searched as the sum of the expectations e^*_D with two vectors which are respectively collinear to the noises $\overline{\overline{v}}_2^2$ and $\overline{\overline{v}}_1^1$. The coefficient is determined by minimum of the quadratic error (OLSQ). Then the exchange rate at time $t=2$ reads:

$$\tilde{e}_2=e^*_D+\psi_0\,\overline{\overline{v}}_2^2+\psi_0\,\overline{\overline{v}}_1^1.$$

This expression shows that the private information of the banks are progressively integrated in the OTC exchange rate and transmitted to the market.

This geometric determination of the OTC exchange rate is carried on until the current time t. The general expression which is obtained by taking away the previous quotes of the quoting bank $\theta(t)$. On the one hand, the Hilbert space H_t^i associated to the information hold by bank $\theta(t)$ at time t can be spanned into the direct sum of the closed Hilbert sub-spaces corresponding to e^*_D and the last B cumulative noises corresponding to the noises of the orders of the quoting bank and the $B-1$ noises integrated in the $B-1$ last OTC exchange rate values $\left(\overline{\overline{v}}_t^{\theta(t)},\overline{\overline{v}}_s^{\theta(s)};\ s=t-(B-1)\text{ to }t-1\right)$:

$$H_t^i = Hv_{D-1} \oplus Hv_{t-(B-1)}^{\theta(t-(B-1))} \oplus \ldots \oplus Hv_t^{\theta(t)}.$$

On the other hand, the OTC exchange rate reads:

$$\tilde{e}_t = E\left[\tilde{e}_D \middle| \mathbf{I}_t^{\theta(t)}\right] = e^*{}_D + \psi_0 \sum_{k=0}^{Max(t-1,\, B-1)} \bar{\bar{v}}_{t-k}^{\theta(t-k)}.$$

2. *Determination of the first difference of the OTC exchange rate*

From the previous equation, the difference $\tilde{e}_t - \tilde{e}_{t-1}$ reads:

$$\tilde{e}_t = \tilde{e}_{t-1} + \psi_0\, \bar{\bar{v}}_t^{\theta(t)} \qquad \text{if } t \le B$$

$$\tilde{e}_t = \tilde{e}_{t-1} + \psi_0 \left(\bar{\bar{v}}_t^{\theta(t)} - \bar{\bar{v}}_{t-B}^{\theta(t)}\right) \quad \text{since } \theta(t-B) = \theta(t) \qquad \text{if } B < t \le B\,P$$

since $\theta(t-B) = \theta(t)$.

These equations are written in the following more compact form:

$$\begin{aligned} &\tilde{e}_t - \tilde{e}_{t-1} = \psi_0\, \tilde{u}_t \\ &\tilde{u}_t = \bar{\bar{v}}_t^{\theta(t)} - H(t-B)\, \bar{\bar{v}}_{t-B}^{\theta(t)} \end{aligned} \qquad t = 1, 2, \ldots, B\,P \qquad \text{(E.1)}$$

where $H(x)$ is the Heaviside function $H(x) = \begin{cases} 1 & x \ge 0 \\ 0 & x < 0 \end{cases}$.

3. *The OTC exchange rate is a random walk*

Recall that :

$$\bar{\bar{v}}_t^i = \sum_{\tau=1}^{t} \tilde{v}_\tau^i, \quad \tilde{v}_\tau^i = \tilde{v}_\tau^{id} / e_{M-1} - \tilde{v}_\tau^{if}$$

where $\tilde{v}_\tau^{id}$, $\tilde{v}_\tau^{if}$ are zero mean and not correlated noises.

The expectations of the noise $\tilde{u}_t$ calculated from its definition (E.1), it writes:

$$E(\tilde{u}_t) = 0, \quad t = 1, 2, \ldots, B\,P$$

The covariance is calculated by using $E\left(\bar{\bar{v}}_t^i\, \bar{\bar{v}}_s^j\right) = \sigma_v{}^2\, \delta_{i,j}\, \min(t, s)$, it reads:

$$E(\tilde{u}_t\, \tilde{u}_s) = \sigma_v{}^2\, \delta_{s,t}\, \min(t, B), \quad t, s = 1, 2, \ldots, B\,P.$$

This results is expected because noises of different banks are independent and the noises of a given bank are not autocorrelated.

The OTC exchange rate is a random walk as soon as $B + 1 \le t \le B\,P$, i. e. when the variance of the noise is the constant $E\left(\tilde{u}_t{}^2\right) = \sigma^2\, B, \quad t = B+1, \ldots, B\,P$.

4. *Comparison between the last quote of the OTC market and the fixing exchange rate*

The last quote is that of the bank B at time $B\,P$; it reads:

$$\tilde{e}_{BP} = e^*{}_D + \psi_0 \left(\bar{\tilde{v}}_{BP}^{B} + \bar{\tilde{v}}_{BP-1}^{B-1} + \ldots + \bar{\tilde{v}}_{BP-(B-1)}^{1} \right)$$

The missing private information to the market correspond to the noises the banks have received since their last quote, that is to say (recall all the private information of bank B is in its last quote $\tilde{e}_{BP}$):

For bank $B-1$: $\tilde{v}_{BP-1}^{B}$.

For bank $B-2$: $\tilde{v}_{BP}^{B-1} + \tilde{v}_{BP-1}^{B-2}$.

.

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For bank 1: $\tilde{v}_{BP}^{1} + \tilde{v}_{BP-1}^{1} + \ldots + \tilde{v}_{BP-(B-2)}^{1}$.

If the number of quoting time is much greater than one $(BP >> 1)$ and if the do not vary very much from one time to another one, the missing private information which are equal to the sum of the what is missing for each bank is close to the daily noise:

$$\bar{\tilde{v}}_{BP}^{B} + \bar{\tilde{v}}_{BP-1}^{B-1} + \ldots + \bar{\tilde{v}}_{BP-(B-1)}^{1} \cong \sum_{i=1}^{B} \bar{\tilde{v}}_{BP}^{i} = \tilde{v}_D$$

Then

$$\tilde{e}_{BP} \to e^*{}_D + \psi_0\ \tilde{v}_D = \tilde{e}_D \qquad \text{if } BP >> 1.$$

5. *Observer's information structure*

At any time, the observer's information structure is the tribe generated by the public information which consists of the past values of the fixing and of the OTC exchange rates, respectively $(\tilde{e}_{D-K};\ 1 \le K \le D-1)$ and $(\tilde{e}_{t-k};\ 1 \le k \le t-1)$. It reads at time $t-1$:

$$\Phi_{t-1} = \sigma\left(\sigma\left(\tilde{e}_{D-K};\ 1 \le K \le D-1\right) \cup \sigma\left(\tilde{e}_{t-k};\ 1 \le k \le t-1\right)\right).$$

According to the assumption that the fixing exchange rate is a linear function of the noises to make the daily noises to be observed, we have been given:

$$\sigma\left(\tilde{e}_{D-K};\ 1 \le K \le D-1\right) \equiv \mathbf{F}\mathrm{I}_{t-1} = \mathbf{I}\mathrm{I}_{t-1}. \qquad \text{(E.1)}$$

On the other hand, the tribe generated by the past values of the OTC exchange rate has to be determined. It reads:

$$\mathbf{I3}_{t-1} \equiv \sigma\left(\tilde{e}_{t-k};\, 1 \le k \le t-1\right).$$

According to the definition of the OTC exchange rate, equation (34) of the main text, $\tilde{e}_{t-1}$ is a linear function of $\tilde{e}_{t-2}$ and of the vector of noises $\tilde{V}_{t-1}^{\theta(t-1)} = \left(\tilde{v}_{t-1}^{\theta(t-1)}, \ldots, \tilde{v}_{\max(t-1,t-1-(B-1))}^{\theta(t-1)}\right)$ received by the quoting bank $\theta(t-1)$ since either its last quote at $t-1-B$ or the opening of the OTC market if it has not been quoting yet. These private information have been conveyed to the market by the quoting bank $\theta(t-1)$ to the market at time $t-1$ by its quote $\tilde{e}_{t-1}$. Then the tribe generated by $\tilde{e}_{t-1}$ reads:

$$\sigma\left(\tilde{e}_{t-1}\right) = \sigma\left(\sigma\left(\tilde{V}_{t-1}^{\theta(t-1)}\right) \cup \sigma\left(\tilde{e}_{t-2}\right)\right) = \sigma\left(\tilde{V}_{t-1}^{\theta(t-1)},\, \tilde{e}_{t-2}\right).$$

Then

$$\mathbf{I3}_{t-1} \equiv \sigma\left(\tilde{e}_{t-k};\, 1 \le k \le t-1\right) = \sigma\left(\tilde{V}_{t-1}^{\theta(t-1)},\, \tilde{e}_{t-2},\, \tilde{e}_{t-k};\, 3 \le k \le t-1\right).$$

This recursive procedure of substitution mainly based on the use of the following relationship of generated tribes by random variables $\sigma(X, Y) = \sigma(\sigma(X) \cup \sigma(Y))$ is carried on until $k = t-1$, to obtain:

$$\mathbf{I3}_{t-1} \equiv \sigma\left(\tilde{e}_{t-k};\, 1 \le k \le t-1\right) = \sigma\left(\sigma\left(\tilde{V}_{t-k}^{\theta(t-k)};\, 1 \le k \le t-1\right) \cup \sigma\left(e^*{}_D\right)\right).$$

The observer, knowing $\left(e_{D-K};\, 1 \le K \le D-1\right)$ gets the information structure (E.1). On the other hand, the expectations even if it is not known by the observer is a linear function of the past values of the daily noises, then according to equation (24) of the main text, it reads:

$$\sigma\left(e^*{}_D\right) = \sigma\left(\tilde{v}_{D-K};\, 1 \le K \le D-1\right) \equiv \mathbf{I1}_{t-1}.$$

Then the information generated by the past values of the quotes is:

$$\mathbf{I3}_{t-1} \equiv \sigma\left(\tilde{e}_{t-k};\, 1 \le k \le t-1\right) = \sigma\left(\sigma\left(\tilde{V}_{t-k}^{\theta(t-k)};\, 1 \le k \le t-1\right) \cup \mathbf{I1}_{D-1}\right).$$

Finally, taking into account which quantities are redundant in among the different tribes, the observer's information structure reads:

$$\Phi_{t-1} = \sigma\left(\sigma\left(\tilde{V}_{t-k}^{\theta(t-k)};\, 3 \le k \le t-1\right) \cup \mathbf{I1}_{D-1}\right) \equiv \mathbf{I3}_{t-1}.$$

that is to say it consists, on the one hand, of the tribe generated by the past values of the quotes because the expectations of the forthcoming fixing is in fact included in the first quote of the first quoting bank, and on the other hand, of the noises, i. e. the private information, which have been conveyed to the market by the banks which have quoted up to time $t-1$.

Coming back to the generating variables, which are the noises relative to the orders, even if they are not observed by the observer, the observer's information structure reads:

$$\Phi_{t-1} = \mathbf{I3}_{t-1} \equiv \sigma\left(\bigcup_{k=1}^{t-1} \sigma\left(\tilde{v}_{t-k}^{\theta(t-k)}, \ldots, \tilde{v}_{\max(1,t-k-(B-1))}^{\theta(t-k)}\right) \cup \sigma\left(\tilde{v}_{D-K};\ K = 1 \text{ to } D-1\right)\right)$$

Then for the observer, the past values of the fixing exchange rate and of the quotes generate all the information relative to the banks' private information.

6. *The conditional statistical properties of the OTC exchange rate: the bank's analysis*

Consider bank i. After time $t-1$, its information structure is given by the knowledge of its private information and of the past quotes; it reads:

$$\mathbf{I}_t^i = \sigma\left(\mathbf{I2}_t^i \cup \mathbf{I3}_t\right)$$

where $\mathbf{I3}_t$ is explicitly given in proposition 8 in the main text.

Consider a *non-quoting bank.* Because of the assumption of statistical independence between the orders of different banks, its private information does not give to the bank any kind of information of the private information of the bank which is going to be quoting at time t. Then this private information is independent of any non-quoting bank's information structure. All the non-quoting banks are in the observer's situation with respect to the noises corresponding to the private information the quoting bank. By contrast, the quoting bank $\theta(t)$ knows its private information which is going to be conveyed to the market by its quote, at time t. Then the banks' conditional properties are given in the following proposition:

PROPOSITION:

1. *The bank's information structure reads:*

$$\mathbf{I}_t^i = \sigma\left(\mathbf{I2}_t^i \cup \mathbf{I3}_t\right) \qquad t = 1, \ldots, B\ P.$$

2. *For bank i, the OTC exchange rate conditional dynamics is characterised by:*

(i) *Its conditional expectations read:*

$$E\left(\tilde{e}_t \middle| \mathbf{I}_{t-1}^i\right) = e_{t-1} + \delta_{i,\theta(t)}\ \psi_0 \sum_{k=0}^{\max(t-1,\ B-1)} \tilde{v}_{t-k}^{\theta(t-k)} \qquad t = 2, \ldots, B\ P$$

(ii) *Its conditional variance reads:*

$$Var\left(\tilde{e}_t \middle| \mathbf{I}_{t-1}^i\right) = \left(1 - \delta_{i,\theta(t)}\right) \psi_0{}^2\ \sigma_v{}^2\ \min(t, B) \qquad t = 2, \ldots, B\ P$$

Then at any time $t > B$, the OTC exchange rate is a martingale.

♦ Part 2, the proof relies on a similar straightforward calculation as it has been done in proposition 8 in the main text. ♦

For the *quoting bank*, as intuitively expected, the OTC exchange rate dynamics is deterministic: it value is given by the last quote of the market and the quoting bank's private information.

B. No bank observes the noises corresponding to its clients' orders

1. *Observer's information structure*

As in the previous case **A**, the observer's information structure reads at time $t-1$:

$$\Phi_{t-1} = \sigma\left(\sigma\left(\tilde{e}_{D-K};\ 1 \le K \le D-1\right) \cup \sigma\left(\tilde{e}_{t-k};\ 1 \le k \le t-1\right)\right).$$

According to the assumption that the fixing exchange rate is a linear function of the noises to make the daily noises to be observed, the information structure due to the past values of the fixing exchange rate is given by equation (E.1).

On the other hand, the tribe generated by the past values of the OTC exchange rate is different because now the OTC exchange rate is determined by an econometric equation. This information structure has to be determined; it is:

$$\sigma\left(\tilde{e}_{t-k};\ 1 \le k \le t-1\right) \equiv \mathbf{F3}_{t-1}.$$

This tribe is determined by using the method presented in the case **A**. According to the definition of the OTC exchange rate, equation (39) of the main text, it is finally obtained:

$$\Phi_{t-1} = \mathbf{F3}_{t-1} \equiv \sigma\left(\tilde{e}_{t-k};\ 1 \le k \le t-1\right) = \sigma\left(\sigma\left(\tilde{U}_{t-k}^{\theta(t-k)};\ 1 \le k \le t-1\right) \cup \mathbf{I1}_{D-1}\right)$$

where for $k = 1$ to $t-1$:

$$\tilde{U}_{t-k}^{\theta(t-k)} = \left(e^{*\theta(t-k)}_{t-k}, \ldots, e^{*\theta(tk1)}_{\max(t-k,\,t-k-(B-1))}, \hat{a}_{t-k}, \hat{b}_{i,t}; i = 0,1,2, \hat{c}\right)$$

By observing the past quotes the observer gets the information which consists of the private information of the bank and its estimators of the OTC exchange rate.

2. *The conditional statistical properties of the OTC exchange rate: the bank's analysis*

For any bank i, whose information structure at time $t = 2, B\ P$ is $\mathbf{F}_t^i = \sigma\left(\mathbf{F2}_t^i \cup \mathbf{F3}_t\right)$, when it is not quoting, because its quotes are not informative of the quoting-bank's private information, then its conditional moments are equal to the observer's ones, particularly the conditional variance $Var\left(\tilde{e}_t \middle| \mathbf{F}_t^i\right) = Var\left(\tilde{e}_t \middle| \Phi_t\right)$.